COSMOLOGY: PRINCIPLES

OBSERVATIONS

LET´S LOOK AROUND US


Barranquilla
Coro
Maracaibo
Caracas
Cartagena
Valledupar
Barquisimeto
Valencia
Sincelejo
Panamá
Panamá
Montería
Mérida
Barinas
Apartadó
Cúcuta
San Cristóbal
Bucaramanga
Venezue
Medellín
Tunja
Parque
Nacional
Natural El
Tuparro
Pereira
Bogotá
Ibagué
Villavicencio
Cali
Colombia
Neiva
Popayán
Google

Our Home The EARTH

Solar System

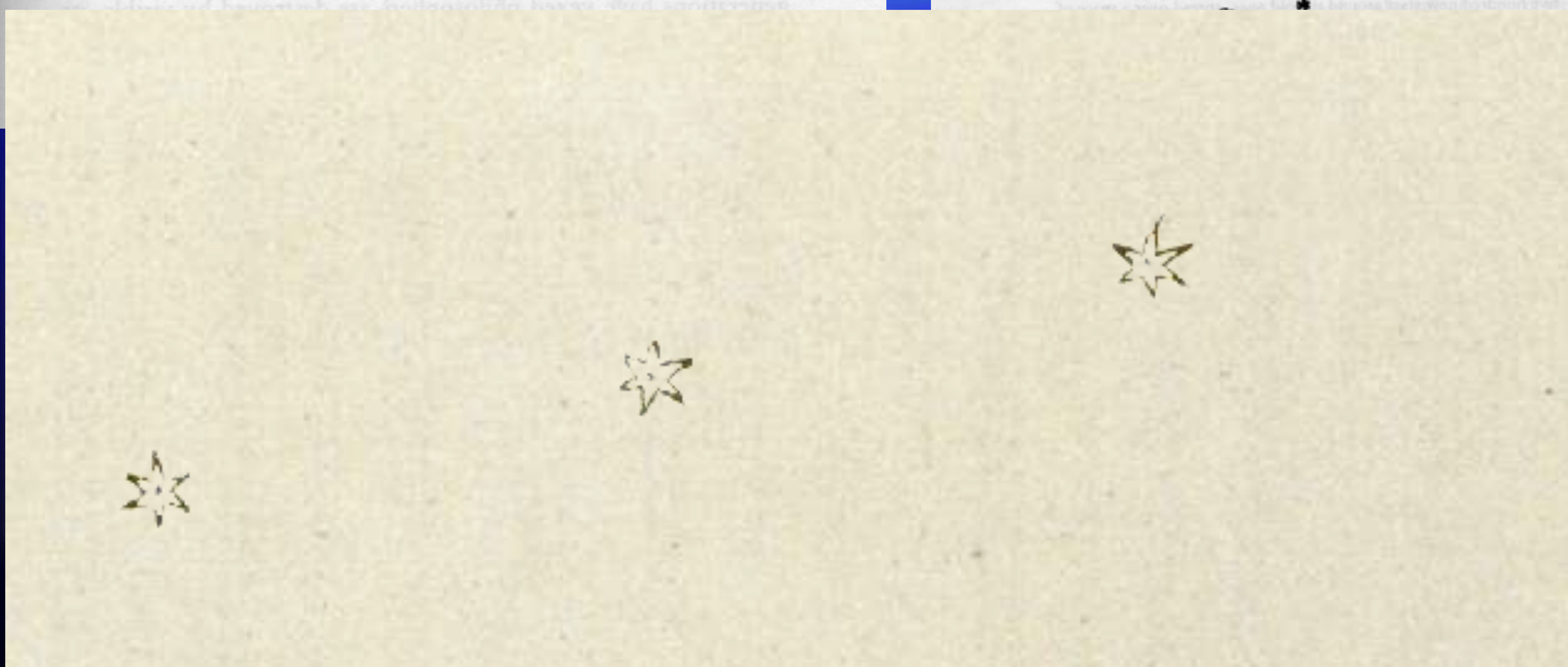
A glowing band along the sky. Have you seen it?

Milky Way in the Sky. What is its nature?

Democritus: Made of Stars 400 BC

Before and after the telescope. 1609

Milky Way is actually made of stars



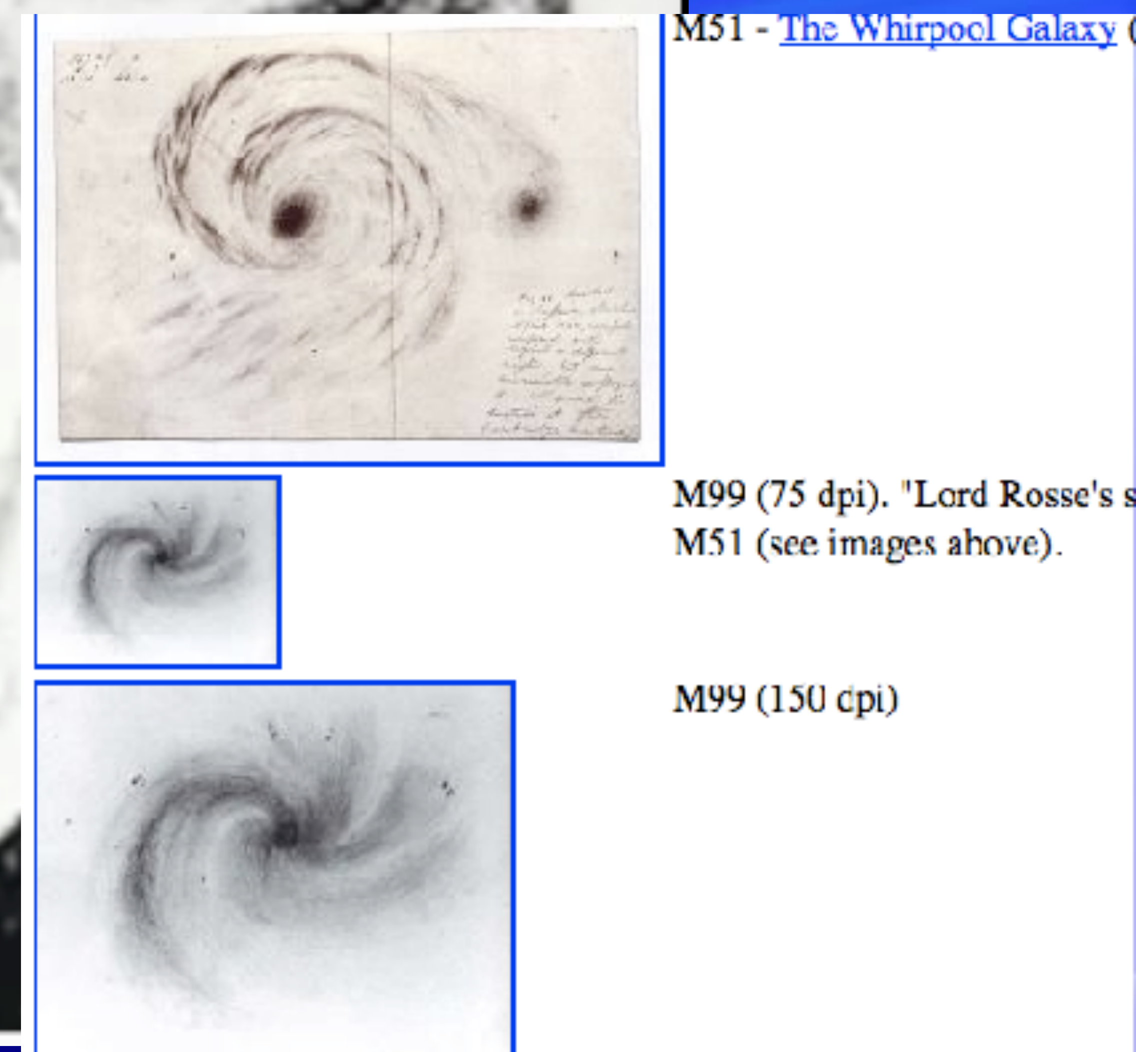
Emmanuel Kant: Island Universes-> Spiral Nebulae and the Milky Way 1707

No propuso

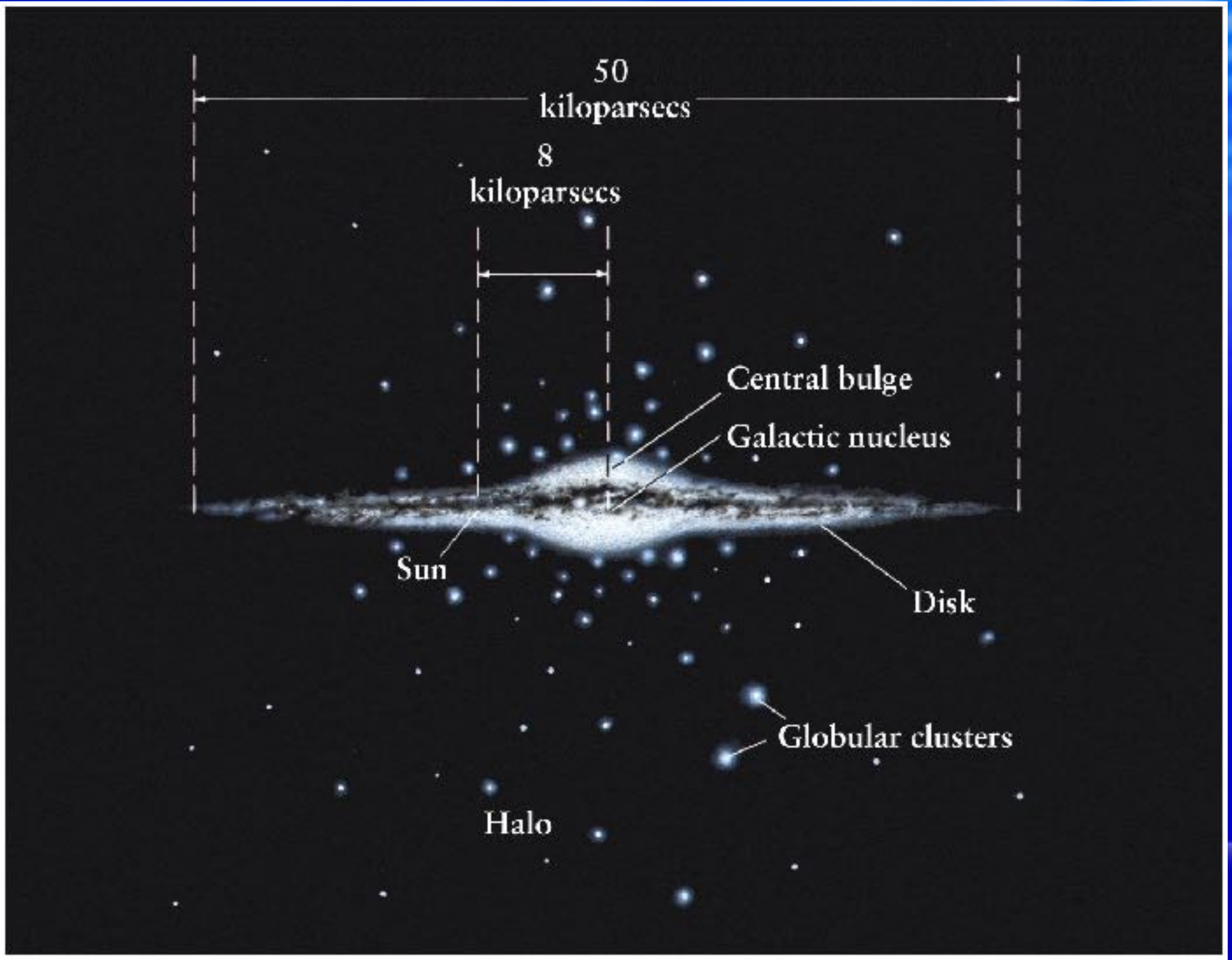
como verificar

la brillante

hipótesis



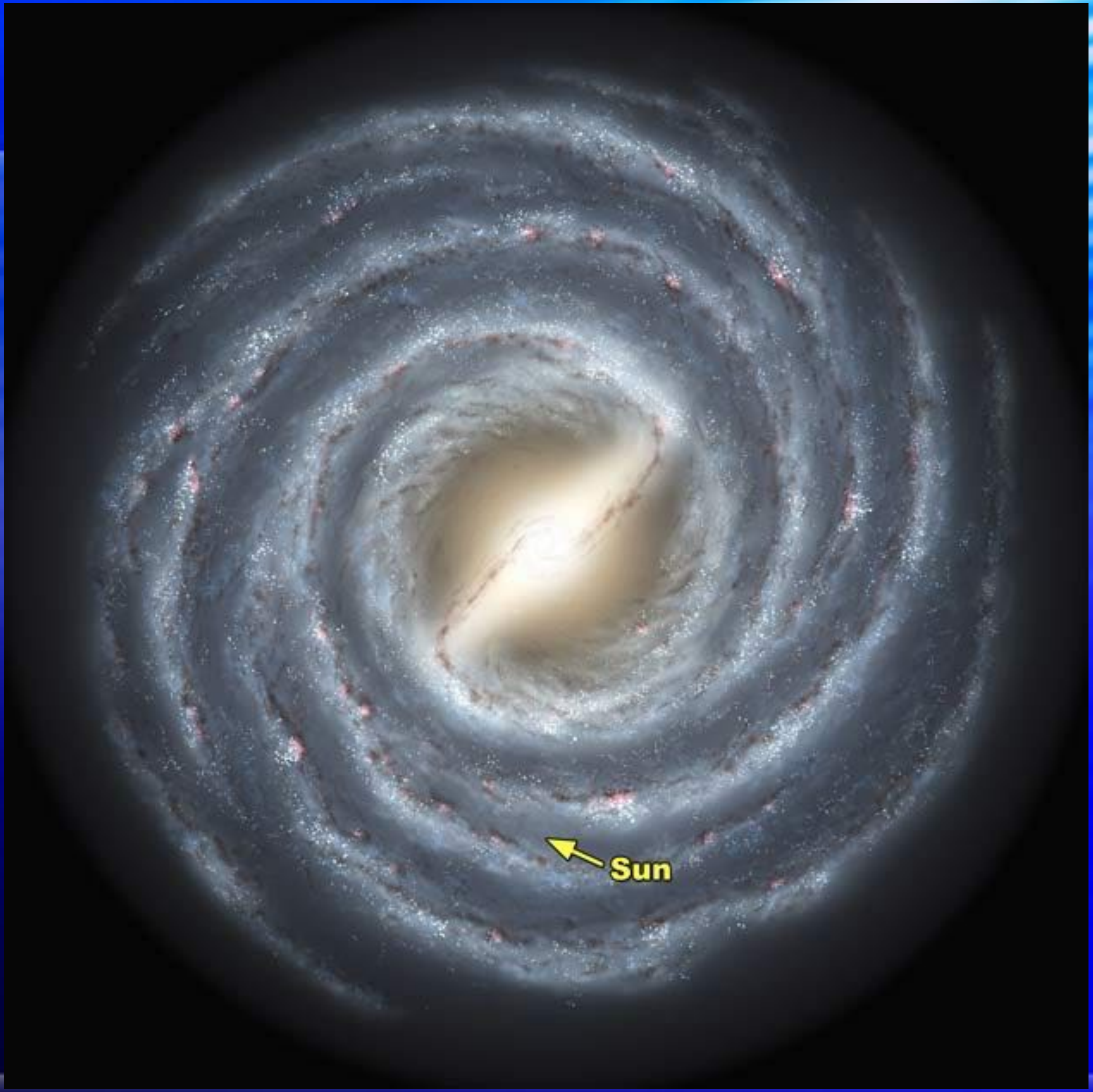
Milky Way (still changing) General Picture



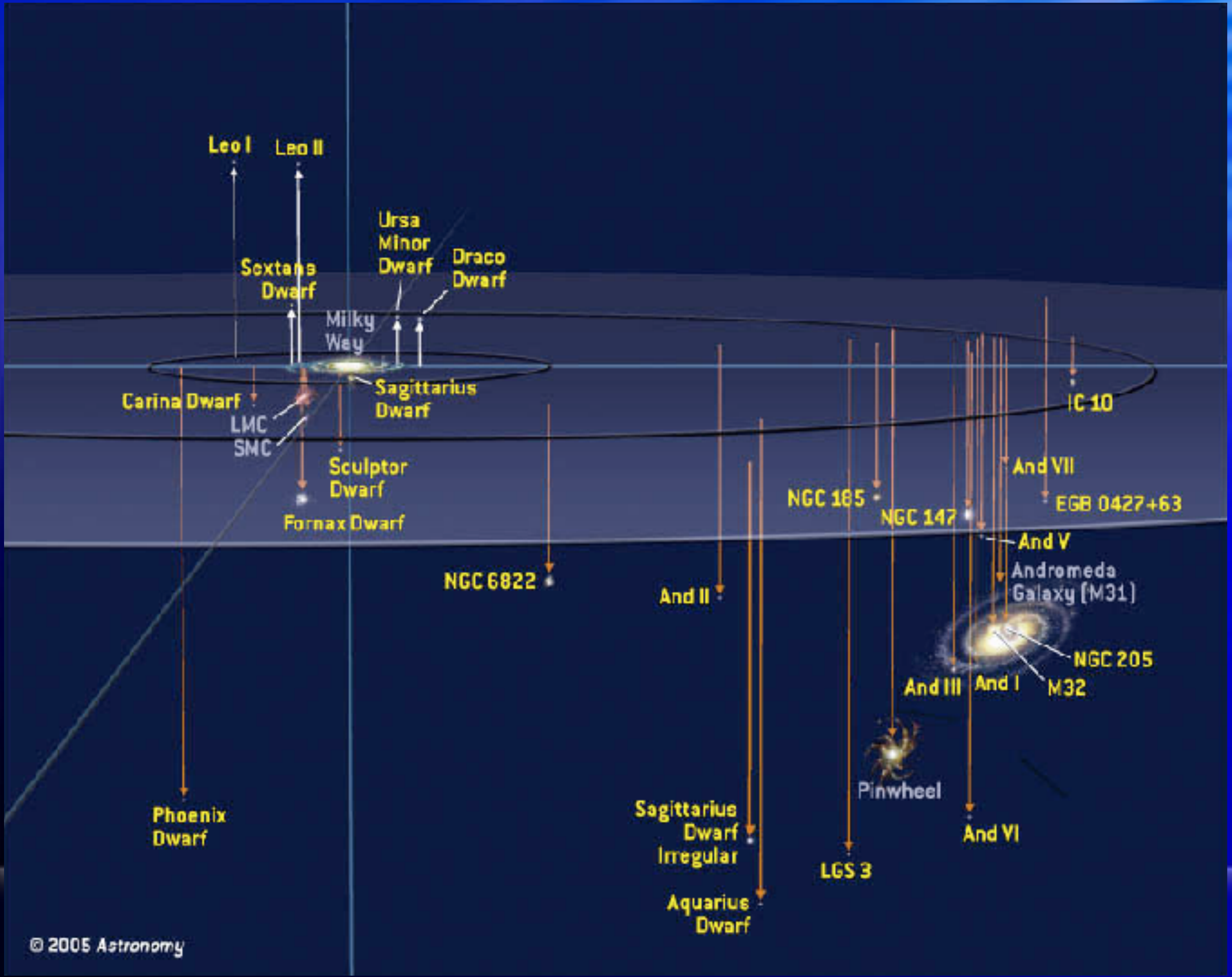
Artístic picture based on observations

Spitzer Space Telescope

(Infrared Light)



¿Are we alone? No
Our Galaxy lives in the so called Local Group



M32 +/- 1

M33-Triangulum

M31-Andromeda M32 and NGC 205

The most common galaxies in the Local Group are Dwarfs

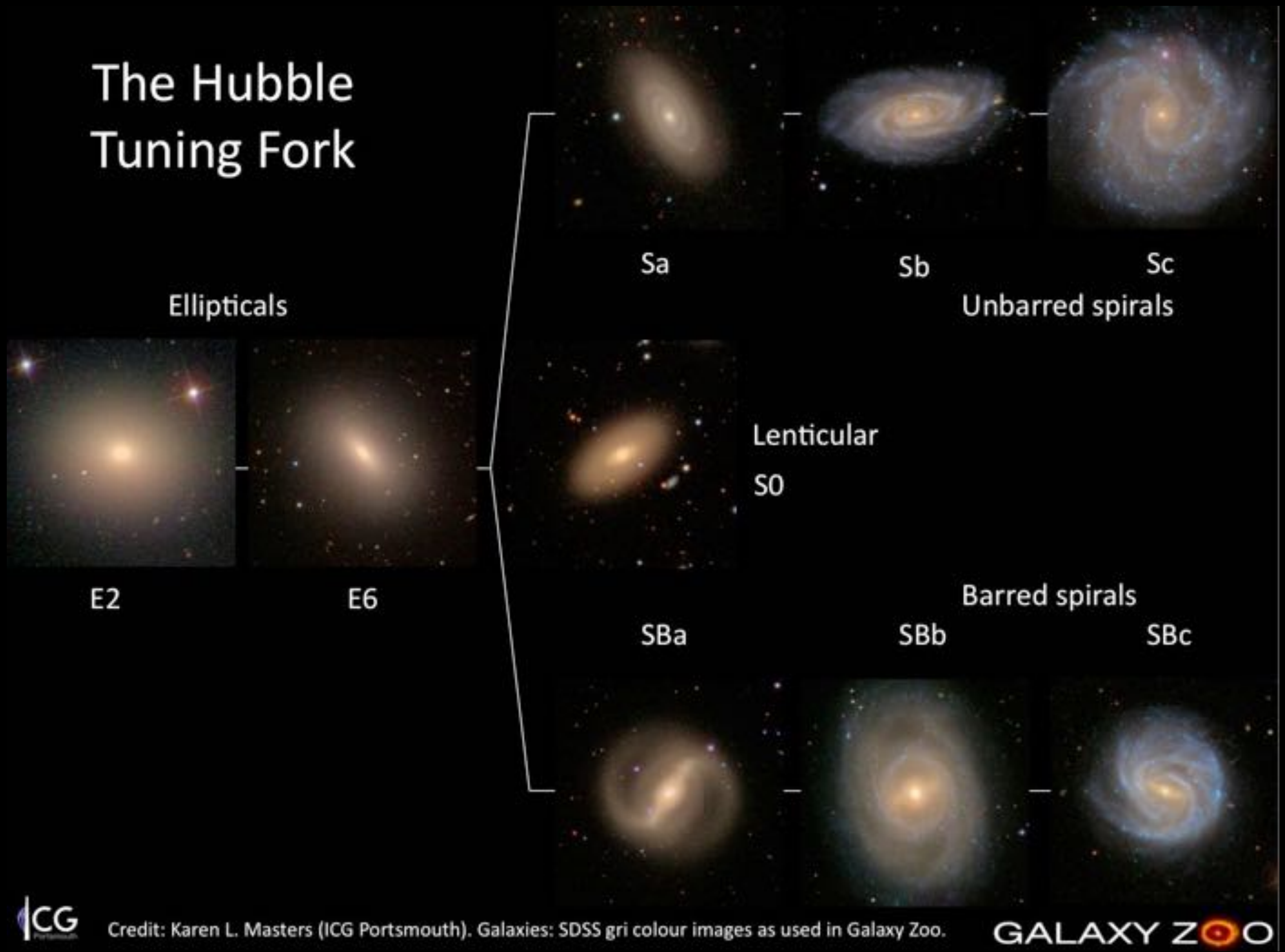
- Irregular
- Spheroidal

E7 or E9?

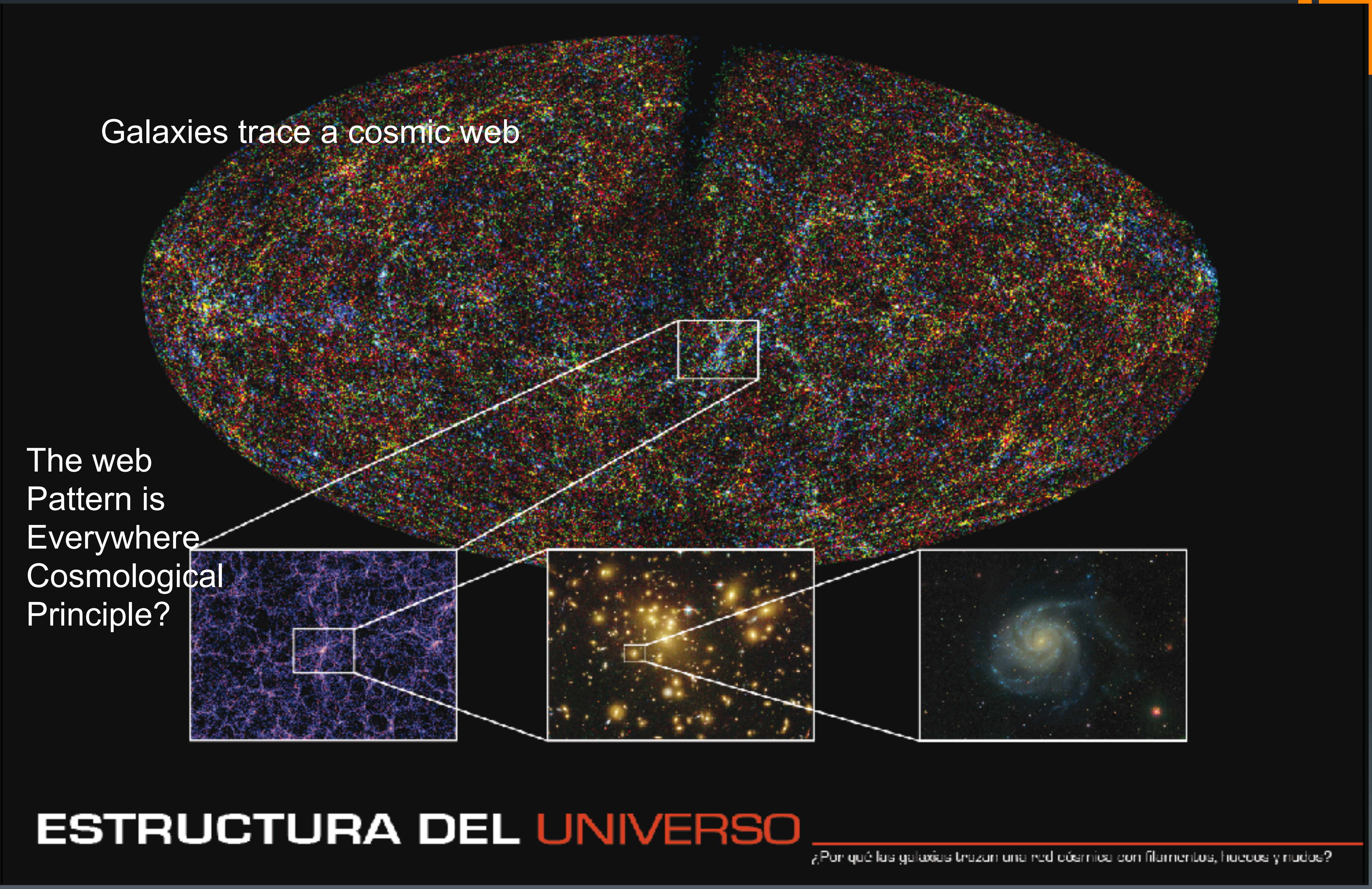
Morphological Hubble Sequence

A Galaxy Zoo

Galaxies are fundamental building blocks of the visible cosmic Large Scale Structure



Each colored dot corresponds to a galaxy in a different cosmic epoch:
Cosmic Structure



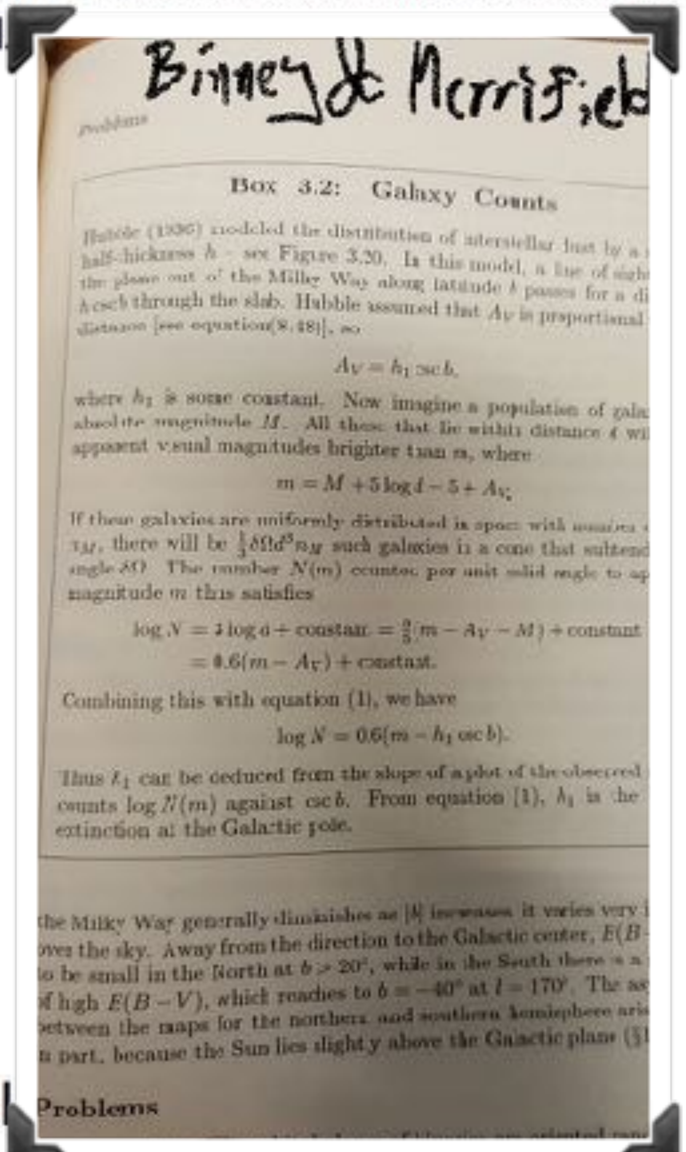
Galaxies trace a cosmic web

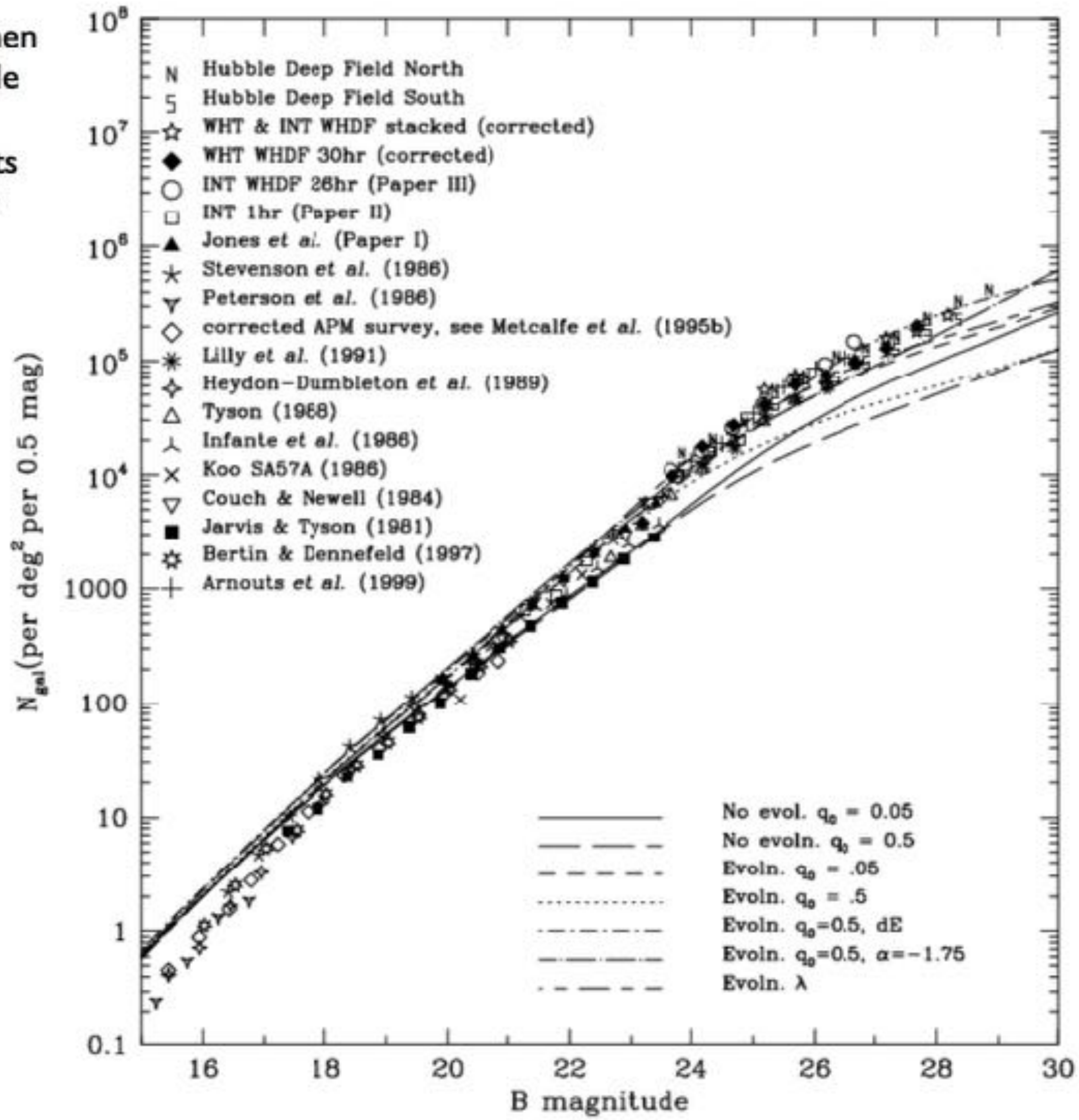
The web
Pattern is
Everywhere
Cosmological
Principle?

Homogeneity of the Universe

Galaxy counts testing Homogeneity

- Deep counts of galaxies provided the first observational arguments for homogeneity of the Universe. If the distribution of galaxies is homogeneous and galaxies are not evolving , then the number of galaxies with apparent magnitude m should scale $10^{0.6m}$. Indeed, this is what is observed up to some magnitude. When counts are too deep, galaxies are preferentially at very large redshifts where the evolution cannot be neglected.





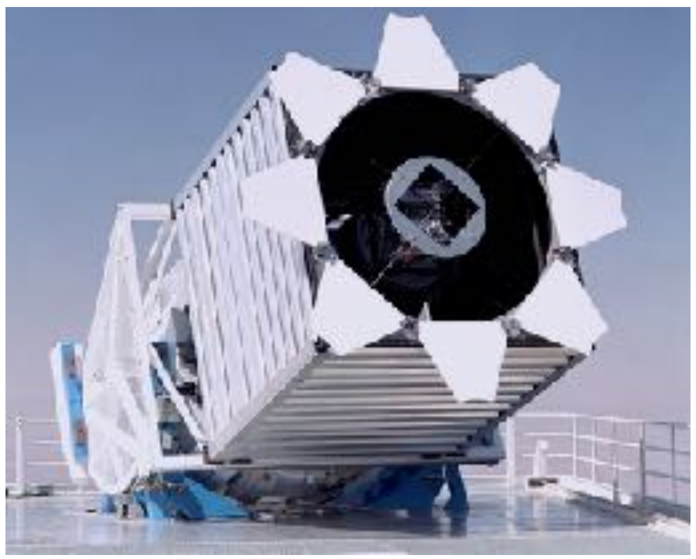
Metcalfe et al

Metcalf etal 2001

- SDSS tested homogeneity of distribution of luminous red galaxies (LRG). Average number-density of LRGs inside a sphere of radius R approaches constant for $R > 30h^{-1}$ Mpc. There are very strong inhomogeneity at smaller scales. Disjoin regions on the sky of size $\sim 2 \times 10^7 h^{-1}\ \mathrm{Mpc}^3$ have variations of 7 percent around the mean density: clear sign of homogeneity on large $\approx 30 - 50h^{-1}$ Mpc scales.

Luminous Red Galaxies

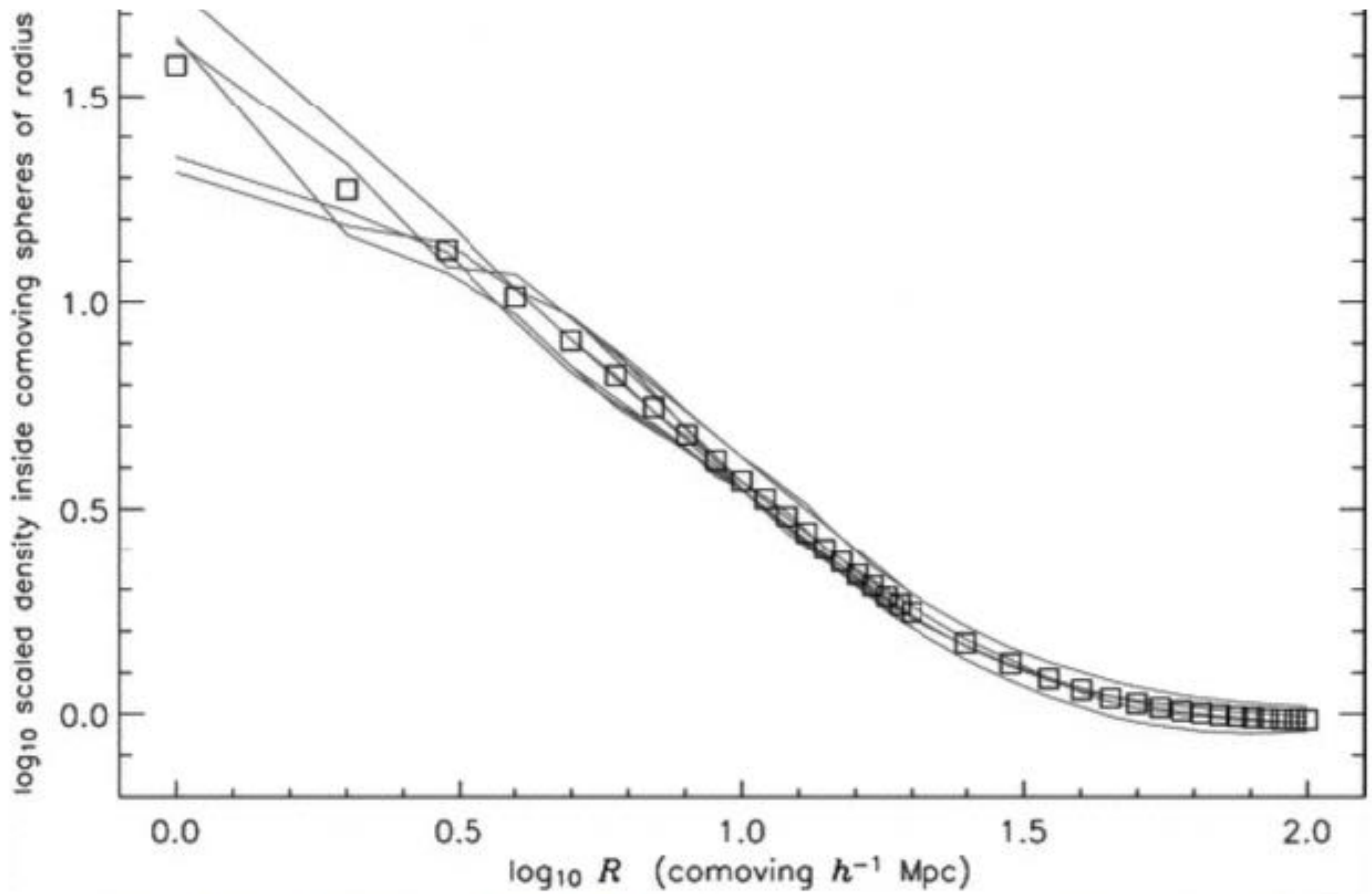
http://classic.sdss.org/dr3/products/general/edr_html/node53.html



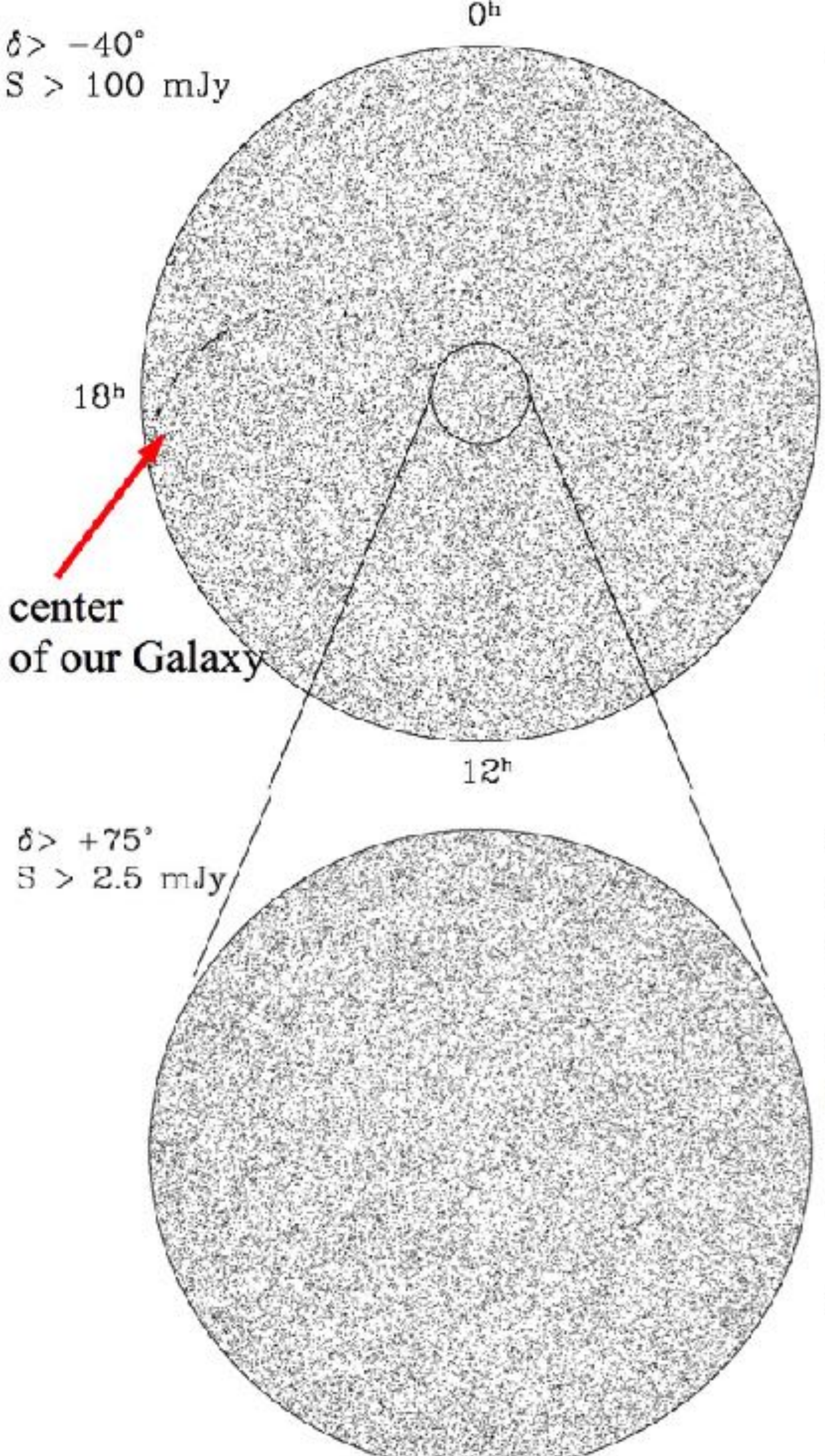
SDSS telescope 2mts

1e6 galaxies with spectra

Hogg et al 2005



Average comoving number density (i.e., number counted divided by expected number from a homogeneous random catalog) of LRGs inside comoving spheres centered on the 3658 LRGs shown in Fig. 1, as a function of comoving sphere radius R. The average over all 3658 spheres is shown with squares, and the averages of each of the five R.A. quantiles are shown as separate lines. At small scales, the number density drops with radius, because the LRGs are clustered; at large scales, the number density approaches a constant, because the sample is homogeneous.



The sky distribution of discrete sources at 1.4 GHz

RG stronger than 100 mJy

• The distribution of discrete sources on the sky is extremely **isotropic**.
=> nearly all RGs in a flux-limited sample are **extragalactic**

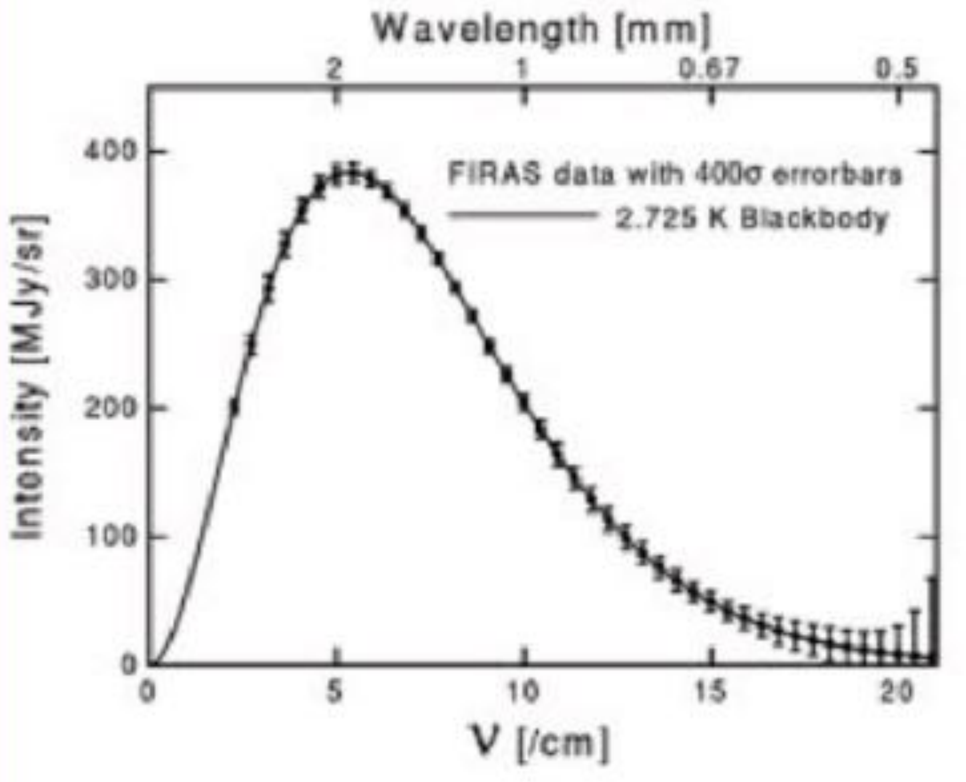
• A similar plot of the brightest galaxies selected at optical or near-infrared wavelengths is **much clumpier** than the radio plot
=> the strongest extragalacticRGs are much farther away than the optically brightest galaxies.
Only 1% of RGs in a flux-limited sample < 100 Mpc

• RGs are at least as clustered as optical galaxies, but the average distance between RGs is much greater than 10 Mpc,
=> their clustering can be detected by sensitive statistical tests.

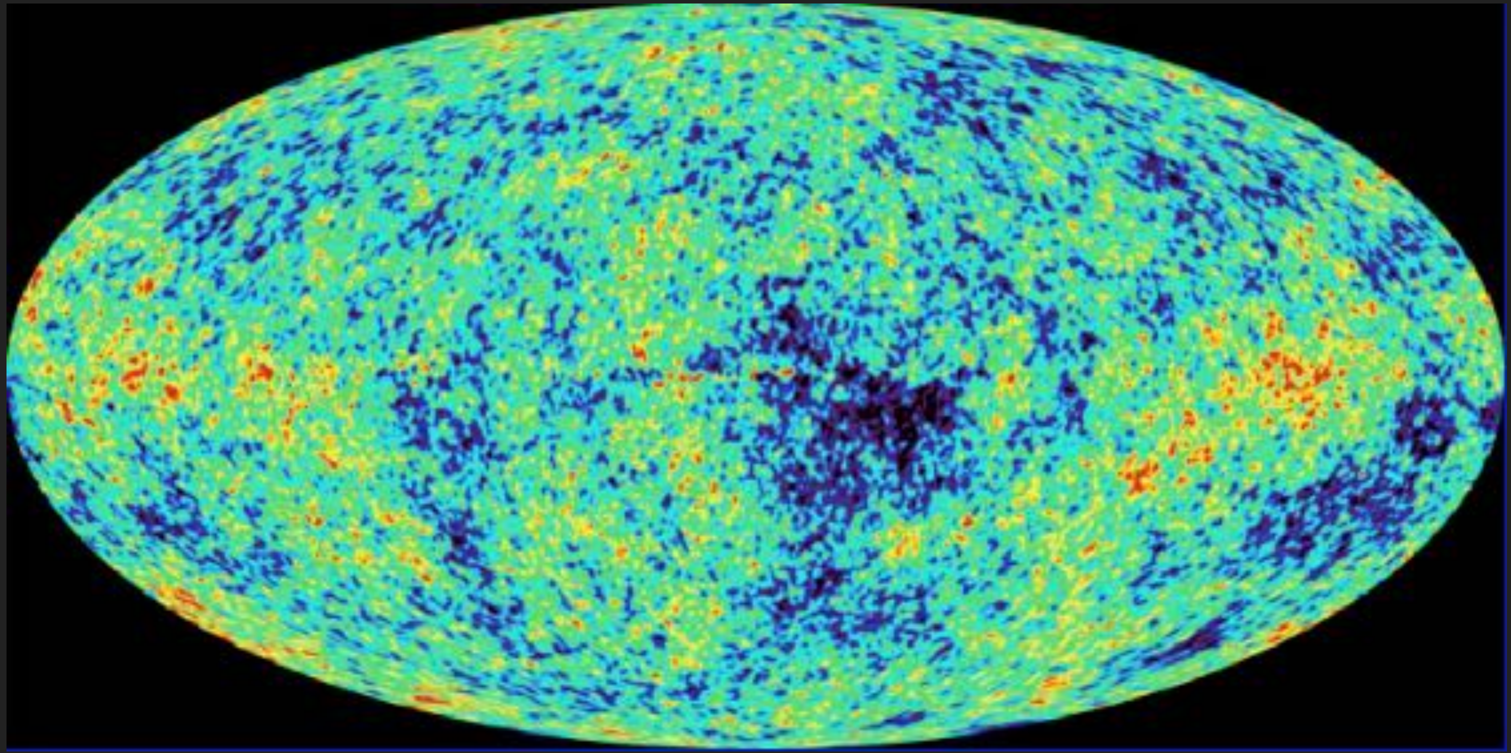
Condon et al.

RG stronger than 2.5 mJy within 15° of the NCP

CMB: MICROWAVE RADIATION WITH BLACK BODY ENERGY DISTRIBUTION IN ALL DIRECTIONS WITH HIGH ACCURACY

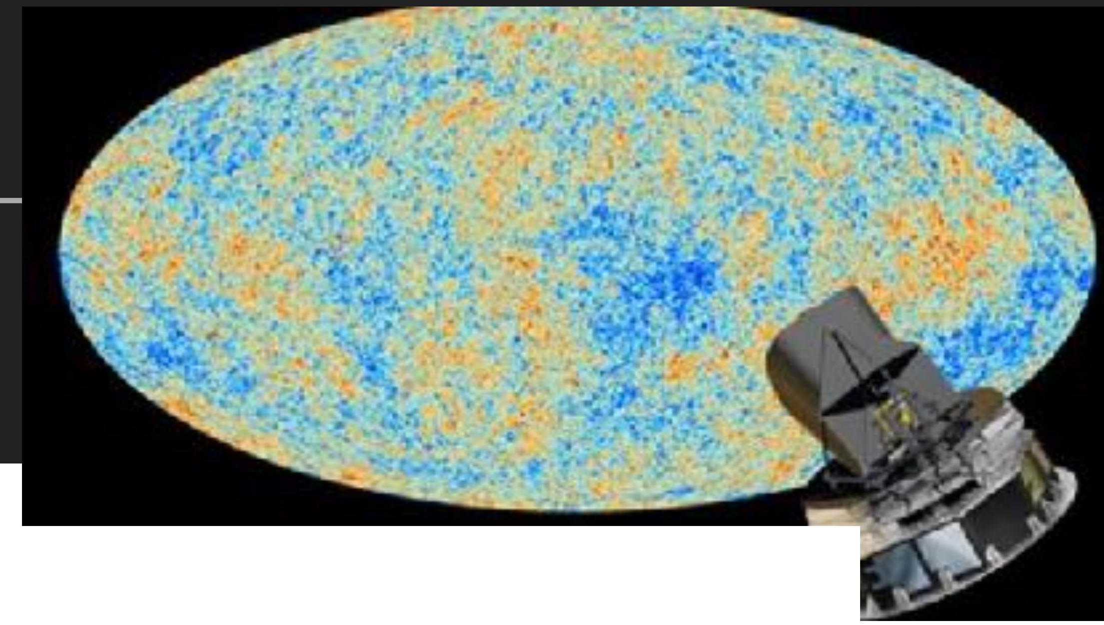


Blackbody spectrum of CMB, measured by COBE (1990)



WMAP

CMB: COSMOLOGICAL PARAMETERS



Planck 2018 results. I.
Overview, and the cosmological legacy of *Planck*

Planck Collaboration: Y. Akrami[59,61], F. Arroja[63], M. Ashdown[69,5], J. Aumont[99], C. Baccigalupi[81], M. Ballardini[22,42], A. J. Banday[99,8], R. B. Barreiro[64], N. Bartolo[31,65], S. Basak[88], R. Battye[67], K. Benabed[57,97], J.-P. Bernard[99,8], M. Bersanelli[34,46], P. Bielewicz[80,8,81], J. J. Bock J. R. Bond[7], J. Borrill[12,95], F. R. Bouchet[57,92 *], F. Boulanger[71,56,57], M. Bucher[2,6], C. Burigana[45,32,48], R. C. Butler[42], E. Calabrese[85], J.-F. Cardoso[57], J. Carron[24], B. Casaponsa[64], A. Challinor[60,69,11], H. C. Chiang[26,6], L. P. L. Colombo[34], C. Combet[73], D. Contreras[21], B. P. Crill[66,10], F. Cuttaia[42], P. de Bernardis[33], G. de Zotti[43,81], J. Delabrouille[2], J.-M. Delouis[57,97], F.-X. Désert[98], E. Di Valentino[67], C. Dickinson[67], J. M. Diego[64], S. Donzelli[46,34], O. Doré[66,10], M. Douspis[56], A. Ducout[57,54], X. Dupac[37], G. Efstathiou[69,60], F. Elsner[77], T. A. Enßlin[77], H. K. Eriksen[61], E. Falgarone[70], Y. Fantaye[3,20], J. Fergusson[11], R. Fernandez-Cobos[64], F. Finelli[42,48], F. Forastieri[32,49], M. Fr

lay[111,11], [27,71,110], het[71,102], [73,80,14]

ABSTRACT

The European Space Agency's *Planck* satellite, which was dedicated to studying the early Universe and its subsequent evolution, was launched on 14 May 2009. It scanned the microwave and submillimetre sky continuously between 12 August 2009 and 23 October 2013, producing deep, high-resolution, all-sky maps in nine frequency bands from 30 to 857 GHz. This paper presents the cosmological legacy of *Planck*, which currently provides our strongest constraints on the parameters of the standard cosmological model and some of the tightest limits available on deviations from that model. The 6-parameter ΛCDM model continues to provide an excellent fit to the cosmic microwave background data at high and low redshift, describing the cosmological information in over a billion map pixels with just six parameters. With 18 peaks in the temperature and polarization angular power spectra constrained well, *Planck* measures five of the six parameters to better than 1 % (simultaneously), with the best-determined parameter (θ_*) now known to 0.03 %. We describe the multi-component sky as seen by *Planck*, the success of the ΛCDM model, and the connection to lower-redshift probes of structure formation. We also give a comprehensive summary of the major changes introduced in this 2018 release. The *Planck* data, alone and in combination with other probes, provide stringent constraints on our models of the early Universe and the large-scale structure within which all astrophysical objects form and evolve. We discuss some lessons learned from the *Planck* mission, and highlight areas ripe for further experimental advances.

(Affiliations can be found after the references)

July 18, 2018

ISOTROPY AND HOMOGENEITY

- Isotropy: Family of triplets (angles: alpha,betha,gamma) wich respect them the whole universe looks the same
- Homogeneity: Family of referente frames (trajectories) wich respect to them the Universe looks identical. **An homogeneous system looks Isotropic for at least 2 observers** (way to test it)
-

HOMOGEINITY TESTED FURTHER? ISOTROPIC AROUND MANY OBSERVERS

- CMB-BlackBody : Another observer Mirror reflects the same CMB Black Body ?

Geocentrism re-examined

Jeremy Goodman*
Princeton University Observatory, Peyton Hall, Princeton, NJ 08544
(15 March 1995)

Abstract

Observations show that the universe is nearly isotropic on very large scales. It is much more difficult to show that the universe is radially homogeneous—that is, independent of distance from us—or equivalently, that the universe is isotropic about distant points. This is usually taken as an axiom, since otherwise we would occupy a special position. Here we consider several empirical arguments for radial homogeneity, all of them based on the cosmic microwave background (CMB). We assume that physical laws are uniform, but we suppose that structure on very large scales may not be. The tightest limits for inhomogeneity on the scale of the horizon appear to be of order ten percent. These involve observations of the Sunyaev-Zel'dovich effect in clusters of galaxies, excitation of low-energy atomic transitions, and the accurately thermal spectrum of the CMB. Weaker limits from primordial nucleosynthesis are discussed briefly.

98.65, 98.80, 95.10, 95.30

TEXTO

$$I_\nu = (1-f)B_\nu(T_0) + fB_\nu(T_\mathrm{r}) \approx B_\nu[(1-f)T_0 + fT_\mathrm{r}] \;+\; O(\Delta T^2), \qquad (1)$$

where $B_\nu(T_0)$ and $B_\nu(T_\mathrm{r})$ are the direct and reflected spectra. We have assumed that the spectrum in any single direction is thermal. The combined spectrum is not, unless $T_\mathrm{r} = T_0$, but it can be approximated by a thermal spectrum to first order in $T_\mathrm{r} - T_0$.

Electron scattering serves as such a mirror. One requires a cluster of galaxies at redshift $z_\mathrm{cl} \sim 1$ with a nonnegligible electron-scattering optical depth, τ. If the cluster fills the telescope beam, the observed spectum summed over polarizations is, for $\tau \ll 1$,

$$I_\nu^\mathrm{obs} = (1-\tau)B_\nu(T_0) + \tau \int \tfrac{3}{4}(1+\cos^2\psi) I'_\nu(\Omega) \frac{d^2\Omega}{4\pi} + y\,\nu^4 \frac{\partial}{\partial\nu}\frac{1}{\nu^2}\frac{\partial}{\partial\nu} B_\nu(T_0), \qquad (2)$$

2nd term test of homogeneity isotropic for them and for us

where $B_\nu(T_0)$ is the unscattered thermal spectrum, obtained from other lines of sight; $(1+z_\mathrm{cl})^3 I'_\nu(\Omega)$ is the specific intensity at the cluster in the direction Ω; and ψ is the scattering angle between this direction and the line of sight. The factor of $(1+\cos^2\psi)$ expresses the angular dependence of electron scattering, summed over polarizations. Similar formulae hold for the individual polarizations, with integrands depending differently on the scattering angles. The third term on the right of Eq. (2) is the Sunyaev-Zel'dovich distortion due to the finite temperature of the electrons ($T_e \gg T_0$): $y \equiv \tau k_B T_e / m_e c^2$ [12]. Since the first and last term have a known dependence on frequency, multifrequency observations can be used to constrain the middle term.

TEXTO

SZ SELECTED CLUSTERS CLOSER THAN CMB BUT LARGER THAN LOCAL SURVEYS

The kSZ effect as a test of general radial inhomogeneity in LTB cosmology

Philip Bull, Timothy Clifton, Pedro G. Ferreira

(Submitted on 10 Aug 2011 (v1), last revised 4 Jan 2012 (this version, v3))

The apparent accelerating expansion of the Universe, determined from observations of distant supernovae, and often taken to imply the existence of dark energy, may alternatively be explained by the effects of a giant underdense void if we relax the assumption of homogeneity on large scales. Recent studies have made use of the spherically-symmetric, radially-inhomogeneous Lemaitre-Tolman-Bondi (LTB) models to derive strong constraints on this scenario, particularly from observations of the kinematic Sunyaev-Zel'dovich (kSZ) effect which is sensitive to large scale inhomogeneity. However, most of these previous studies explicitly set the LTB 'bang time' function to be constant, neglecting an important freedom of the general solutions. Here we examine these models in full generality by relaxing this assumption. We find that although the extra freedom allowed by varying the bang time is sufficient to account for some observables individually, it is not enough to simultaneously explain the supernovae observations, the small-angle CMB, the local Hubble rate, and the kSZ effect. This set of observables is strongly constraining, and effectively rules out simple LTB models as an explanation of dark energy.

Comments:	14 pages, 9 figures. Minor update to match the published version (includes typo correction in Eq. 16)
Subjects:	**Cosmology and Nongalactic Astrophysics (astro-ph.CO)**; General Relativity and Quantum Cosmology (gr-qc)
Journal reference:	Phys. Rev. D 85, 024002 (2012)
DOI:	10.1103/PhysRevD.85.024002
Cite as:	arXiv:1108.2222 [astro-ph.CO]
	(or arXiv:1108.2222v3 [astro-ph.CO] for this version)

Strongly limits the possibility of an inhomogenous Lemeatre Bondi Tolman universe

Probing cosmological isotropy with Planck Sunyaev-Zeldovich galaxy clusters

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[3]APC, AstroParticule et Cosmologie, Universite Paris Diderot, 75205 Paris Cedex 13, France

Isotropy test too

1 December 2015

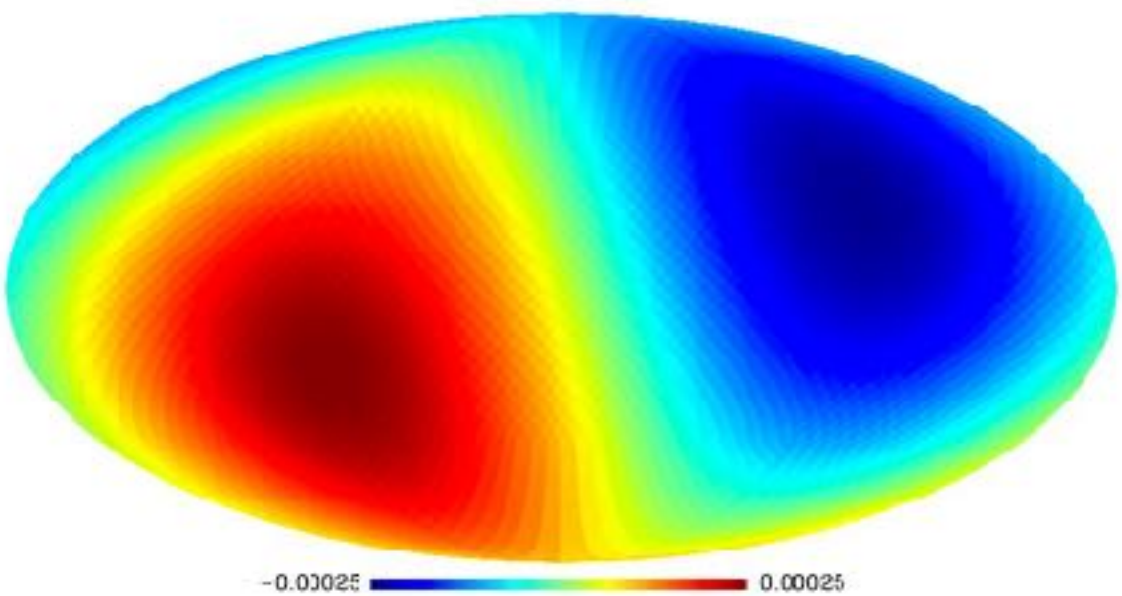
ABSTRACT

We probe the hypothesis of cosmological isotropy using the Planck Sunyaev-Zeldovich (PSZ) galaxy clusters data set. Our analyses consist on a hemispherical comparison of the clusters angular distribution, searching for a preferred direction in the large-scale structure of the Universe. We obtain a maximal dipolar signal at the direction $(l, b) = (53.44°, 41.81°)$ whose antipode points toward $(l, b) = (233.44°, -41.81°)$. Interestingly, this antipode is marginally consistent with the anomalous Cold Spot found in the Cosmic Microwave Background, located at $(l, b) \simeq (209°, -57°)$, which might be possibly aligned with a supervoid at $z \sim 0.2$ with ~ 200 Mpc/h of radius. The statistical significance of this result is assessed with ensembles of Monte Carlo realisations, finding that only a small number of runs are able to reproduce a close direction to this one, hence rejecting the null hypothesis of such direction being a random fluctuation of the data. Moreover, the PSZ catalogue presents a mild discrepancy with the isotropic realisations unless we correct some effects, such as the non-uniform exposure function of Planck's observational strategy, on the simulated data sets. We also perform a similar analysis to a smaller, albeit optimised sub-sample of PSZ sources, finding a better concordance with isotropic realisations, yet no correlation with the supervoid is obtained this time. Thus, we conclude that the dipole anisotropy found on galaxy clusters angular distribution can be partially attributed to an anomalous feature in the large-scale structure, though the significance of this result is sufficiently reduced when corrections to systematic effects are taken into account.

Key words: Cosmology: Observations; large-scale structure of Universe; galaxy clusters

TEXTO

Sl -> rms deviation from mean number density of clusters at different angular scales



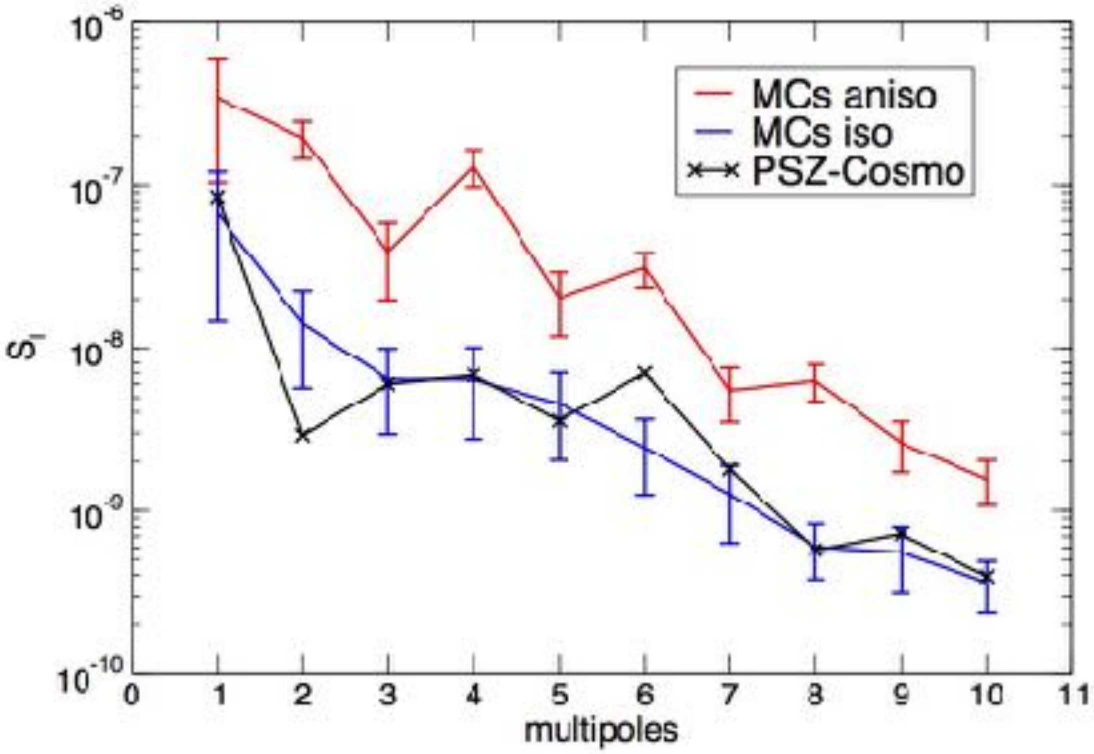
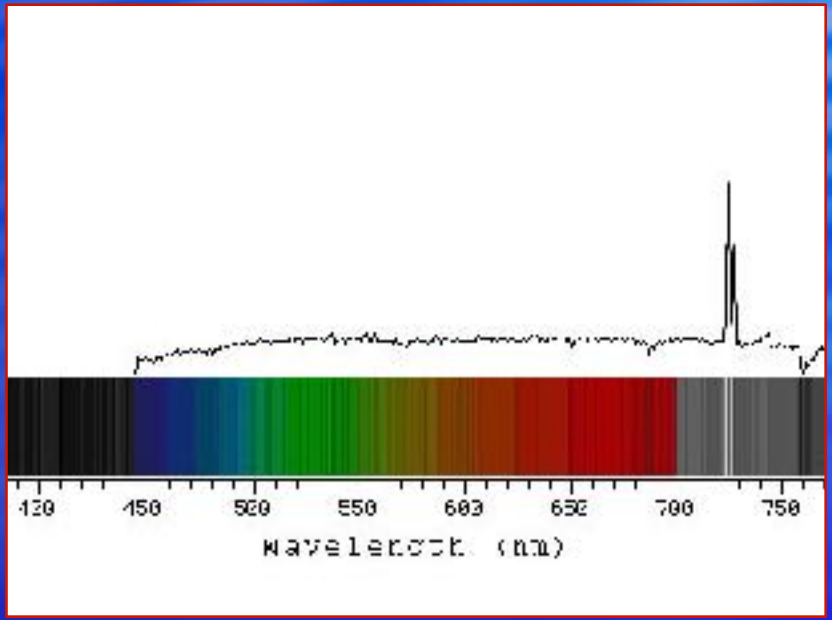


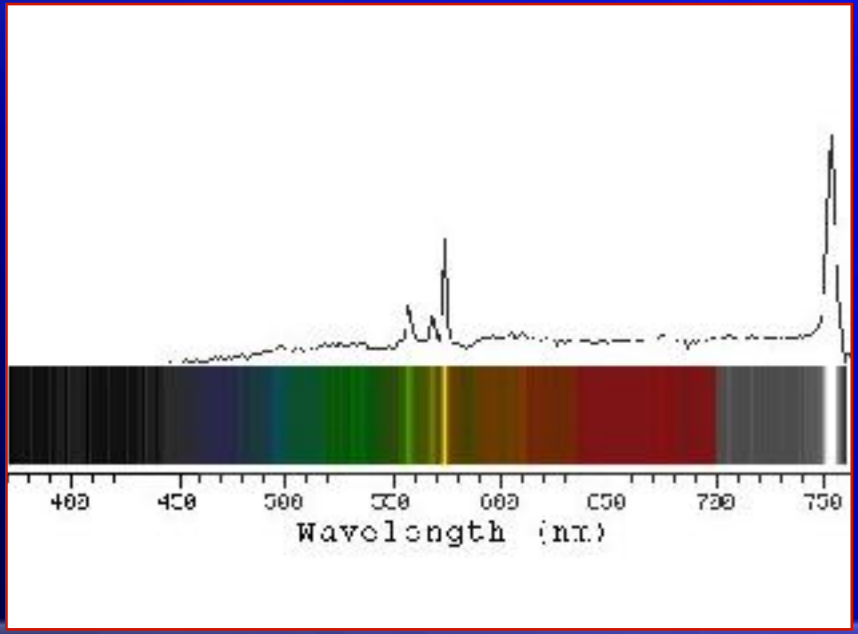
Figure 5. *Left panel*: The dipole-only contribution of the sigma-map of the PSZ-cosmo sub-sample (Figure 4, right panel). *Right panel*: The angular power spectrum, $\{S_\ell\}$, obtained from the sigma-map of the PSZ-cosmo sub-sample (Figure 4, right panel), compared with the angular power spectra from the isotropic and anisotropic MC ensembles.

Galaxies velocities along the line of sight

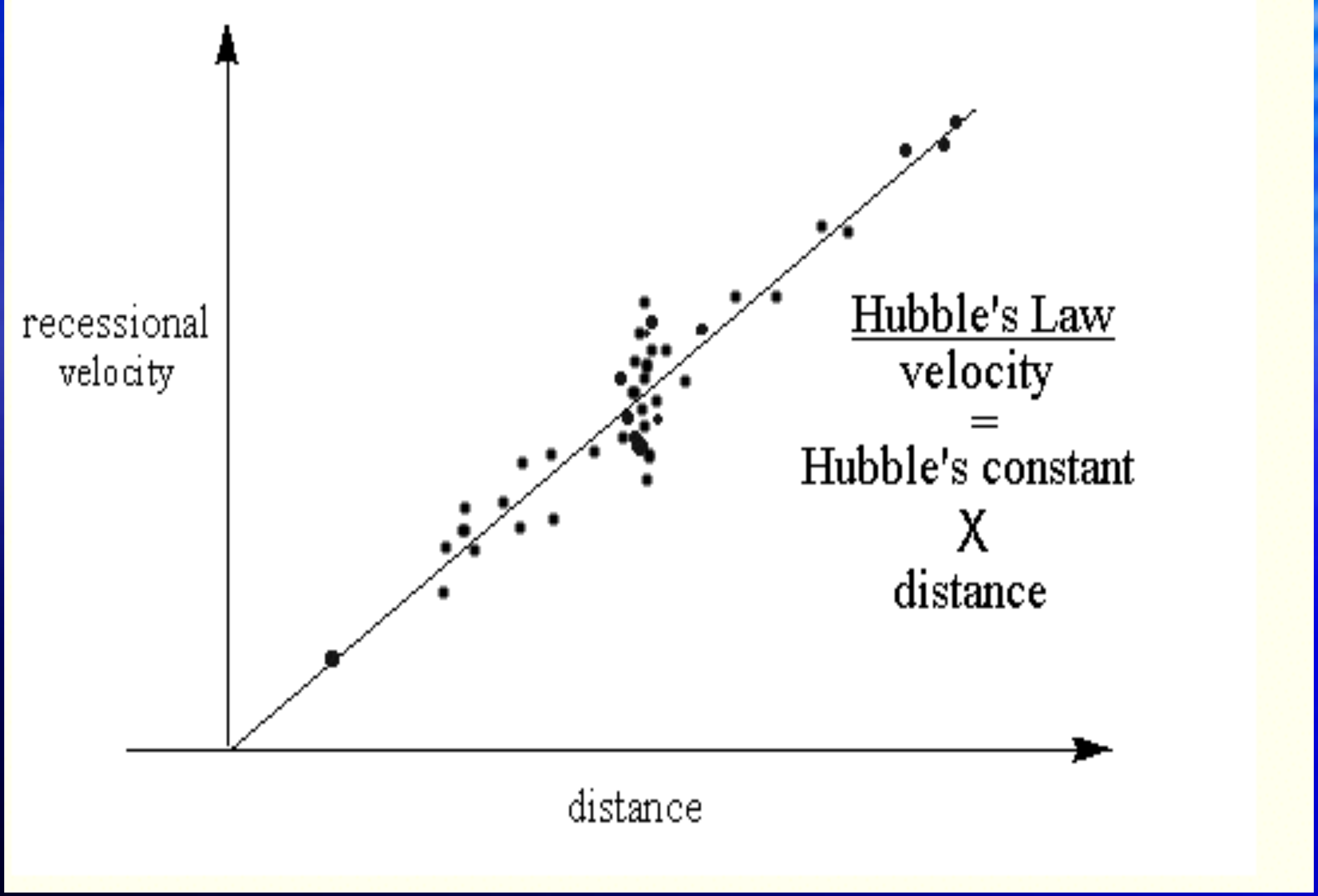
Galaxia KUG 1217



Galaxia IRAS F09159

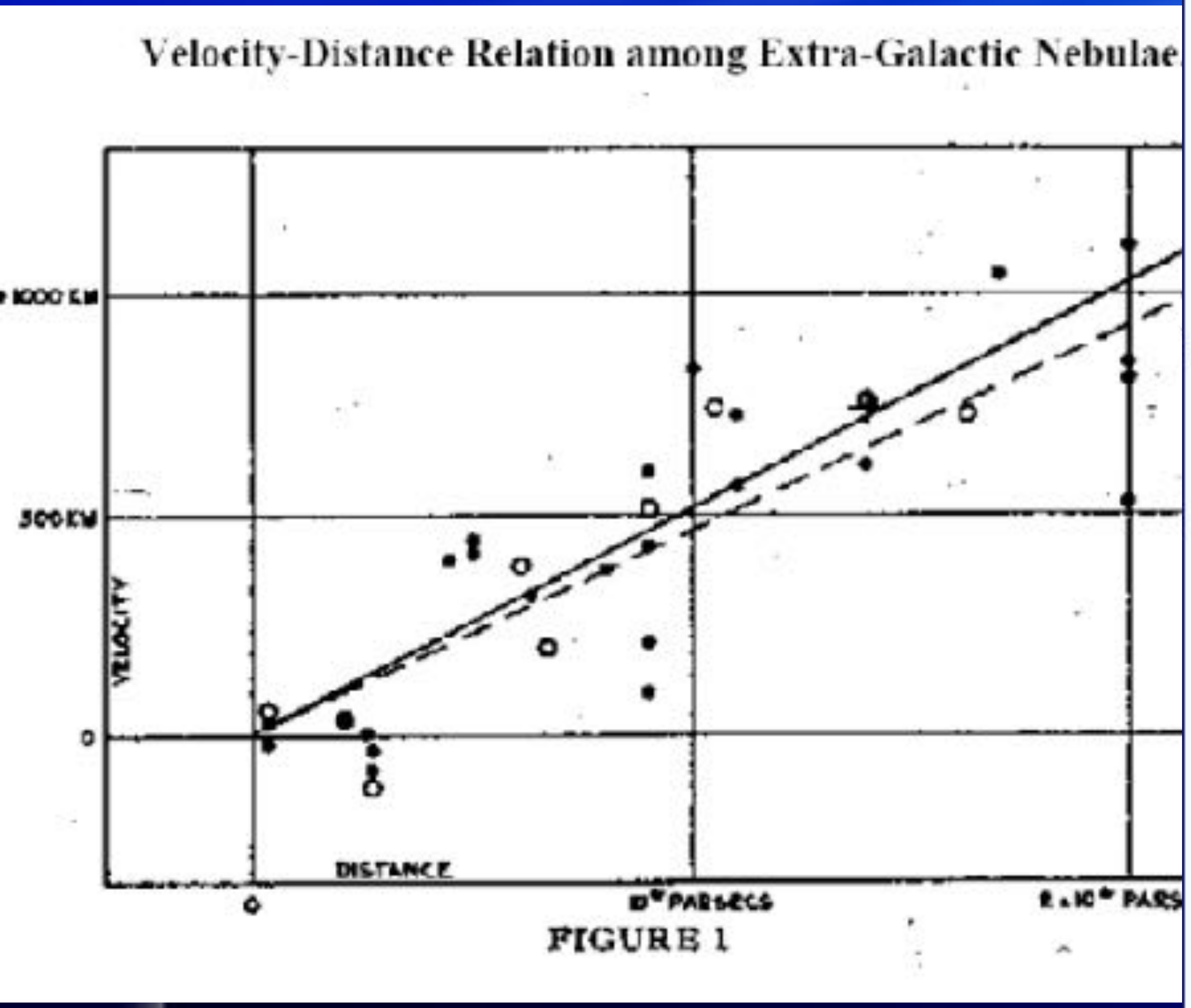


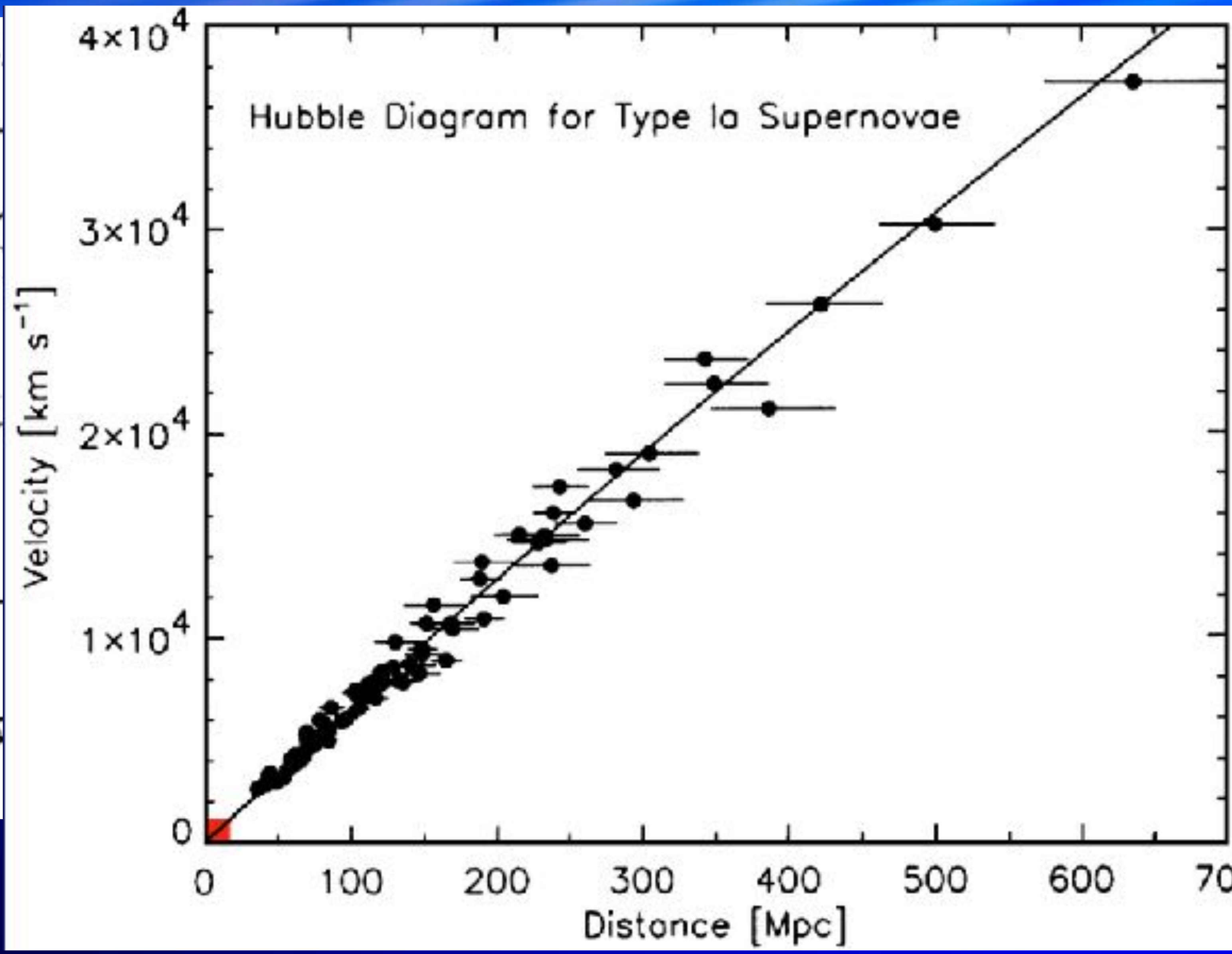
Hubble Law (as farther a galaxies is it moves away faster, Physics?)



Same result along all directions and distances

Linear Relation V=Hd
Regardless of the reference galaxy because of the functional form





Original

Hubble

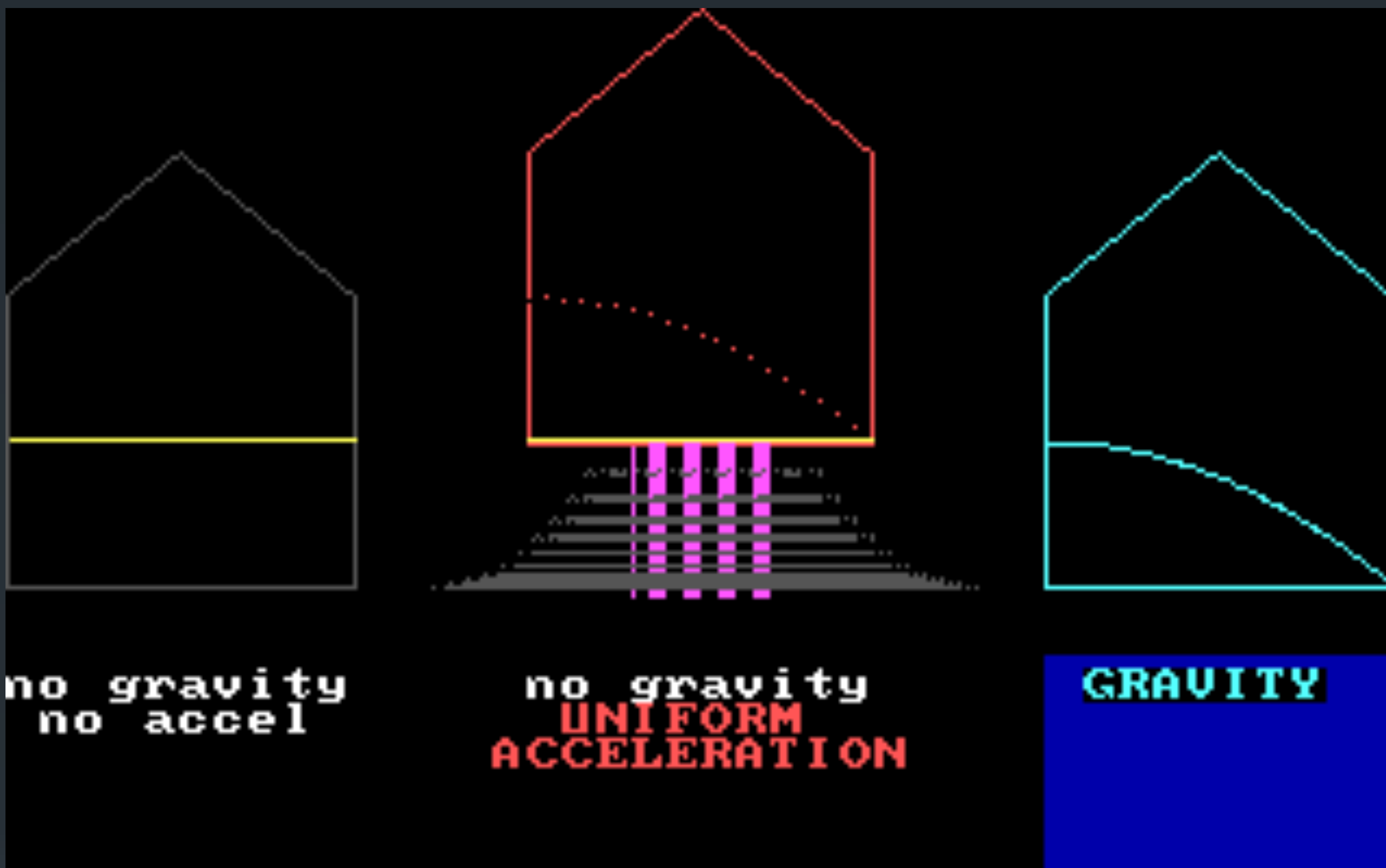
FUNDAMENTAL FORCE DRIVING UNIVERSE EVOLUTION?

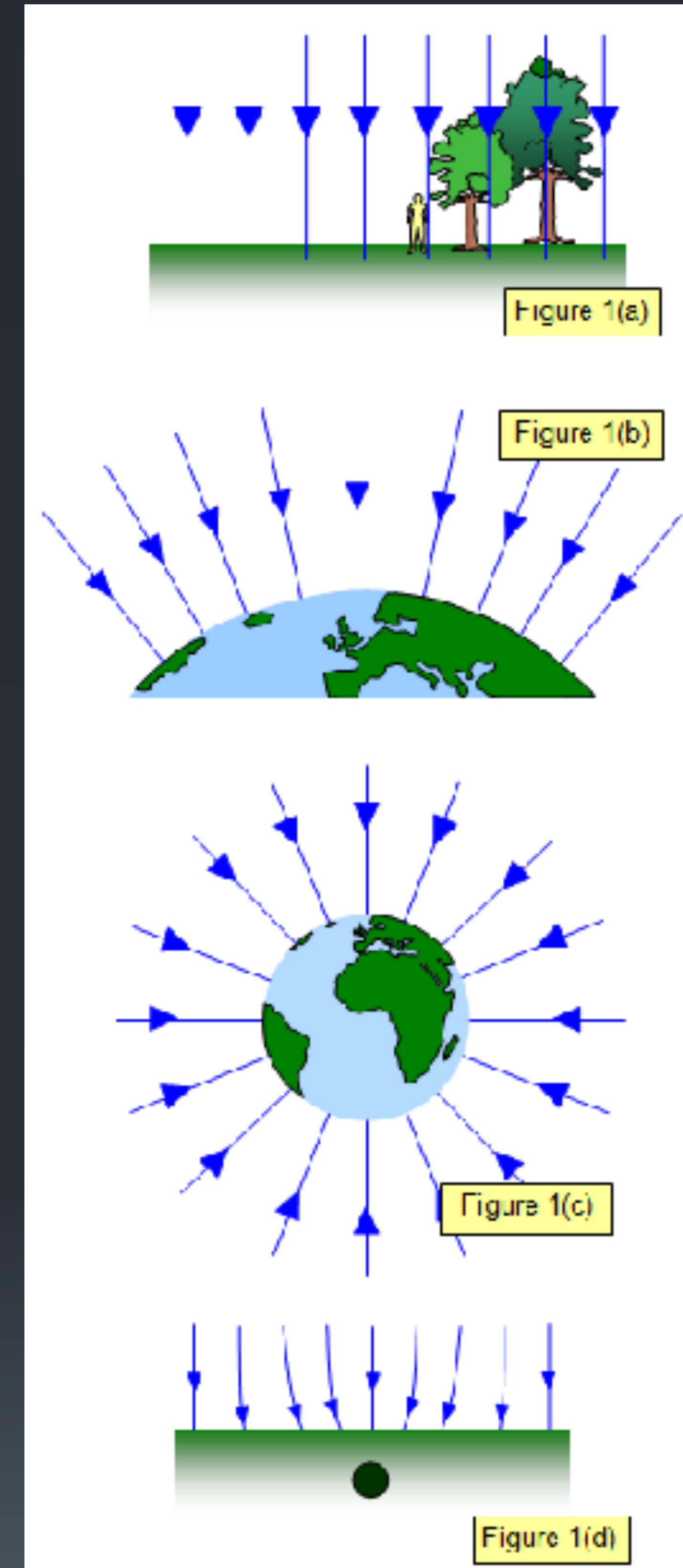
- Color/Strong: Confinement inside nucleons, short range
- Weak: Short Range
- Electromagnetic: Ionized Universe. CMB Black Body spectrum limits, different from synchrotron
- Gravitacional

ELEMENTS NEEDED TO BUILD A UNIVERSE MODEL

- Evolution driven by Gravity
- Observer independent description: General Relativity (GR)
- Homogeneous and Isotropic: observational constraints. Defines GR Metric
- something else?

Equivalence Principle





$$G_{uv} = 8\pi T_{uv}$$

Einstein Equation, compare with Poisson equation

left term curvature from metric derivation

right term sources: energy, density, pressure

Some Corrections Geodesic deviation

$$\frac{d^2 x^\alpha}{du^2} + \Gamma^\alpha_{\beta\gamma} \frac{dx^\beta}{du} \frac{dx^\gamma}{du} = 0$$

WE NEED TO SOLVE EINSTEIN EQUATION

- We must include the Cosmological Principle Homogeneity and Isotropy
- We have 3 spatial dimension and 1 temporal
- G tensor is an object that requires the metric: invariant distance in an arbitrary space

(C) **Spherically Symmetric Homogeneous Space-Time.** Suppose that the dimensionality of the whole space-time is $N = 4$, that three of the eigenvalues of its metric are positive and one is negative, and that it has maximally symmetric *three*-dimensional subspaces whose metric has positive eigenvalues and arbitrary curvature. Then there is one v-coordinate and three u-coordinates, and (13.5.27) gives

$$-d\tau^2 = g(v)\,dv^2 + f(v)\left\{d\mathbf{u}^2 + \frac{k(\mathbf{u}\cdot d\mathbf{u})^2}{1 - k\mathbf{u}^2}\right\}$$

where $f(v)$ is a positive function and $g(v)$ is a negative function of v. It is very convenient to define new coordinates t, v, θ, φ by

justification?

$$\int (-g(v))^{1/2}\,dv \equiv t$$

Weinberg, Gravitation & Cosmology

$$u^1 \equiv r \sin\theta \cos\varphi$$
$$u^2 \equiv r \sin\theta \sin\varphi$$
$$u^3 \equiv r \cos\theta$$

We then have

$$d\tau^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1 - kr^2} + r^2\,d\theta^2 + r^2 \sin^2\theta\,d\varphi^2\right\} \qquad (13.5.32)$$

where $R(t) \equiv \sqrt{f(v)}$.

The first two examples show how it is possible to capture the essence of spherical symmetry by giving a qualitative description of a space or space-time in terms of dimensionalities, signs of eigenvalues and curvatures, and the maximal

TE

0

2 The Robertson-Walker Metric

The formulation of the Cosmological Principle given in the last section allows us to apply the results of Section 13.5 for spaces with maximally symmetric subspaces. We see immediately that it must be possible to choose coordinates r, θ, ϕ, t, for which the metric takes the form given in Eq. (13.5.32):

justification?

$$d\tau^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2\,d\theta^2 + r^2\sin^2\theta\,d\phi^2\right\} \qquad (14.2.1)$$

where $R(t)$ is an unknown function of time, and k is a constant, which by a suitable choice of units for r can be chosen to have the value $+1$, 0, or -1. (These are not necessarily the same as the cosmic standard coordinates introduced in the last section, although t in Eq. (14.2.1) is the cosmic standard time, or a function of it.) The metric (14.2.1) is known in cosmology as the *Robertson-Walker metric*.

It is interesting to consider the geometrical properties of the three-dimensional spaces of constant t. These have metric

$${}^3g_{rr} = \frac{R^2(t)}{1-kr^2} \qquad {}^3g_{\theta\theta} = r^2R^2(t) \qquad {}^3g_{\phi\phi} = r^2\sin^2\theta R^2(t) \qquad (14.2.2)$$

with ${}^3g_{\mu\nu}$ vanishing for $\mu \neq \nu$. Comparing with (13.3.23)–(13.3.25) shows that the *three-dimensional* curvature scalar is

$${}^3K(t) = kR^{-2}(t) \qquad (14.2.3)$$

For $k = -1$ or $k = 0$ the space is infinite, while for $k = +1$ it is finite (though unbounded), with proper circumference given by Eq. (13.3.33) as

$${}^3L = 2\pi R(t) \qquad (14.2.4)$$

and proper volume given by Eq. (13.3.29) as

$${}^3V = 2\pi^2R^3(t) \qquad (14.2.5)$$

For $k = +1$ the spatial universe can be regarded as the surface of a sphere of radius $R(t)$ in four-dimensional Euclidean space (see Section 13.3), and $R(t)$ can justly be called the "radius of the universe." For $k = -1$ and $k = 0$ no such

Cosmology in a Nutshell...

General Relativity

Einstein's Field Equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Cosmological Principle

Universe is homogeneous & Isotropic

Riemannian Geometry

Friedmann-Robertson-Walker Metric

$$\mathrm{d}s^2 = a^2(\tau)\left[\mathrm{d}\tau^2 - \mathrm{d}\chi^2 - f_K^2(\chi)\,\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)\right]$$

Friedmann Equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

i.e. spatial terms only. $g_{0i} = 0$ because isotropy $\implies$ no preferred direction (cf Schwarzschild). For dl^2 we look for a 3D-space of constant curvature, analagous to the surface of a sphere.

Consider the surface of a sphere in Euclidean 4D. Using Cartesian coordinates (x, y, z, w), but replacing (x, y, z) by spherical polars (ρ, θ, ϕ), we have

$$dl^2 = d\rho^2 + \rho^2 \, d\Omega^2 + dw^2,$$

Using ρ for the radial coord, to save r for later; see below.

where $d\Omega^2$ is short-hand for the angular terms. Also

$$x^2 + y^2 + z^2 + w^2 = \rho^2 + w^2 = R^2,$$

and so

$$\rho \, d\rho + w \, dw = 0.$$

Therefore

$$dw^2 = \frac{\rho^2 \, d\rho^2}{w^2} = \frac{\rho^2 \, d\rho^2}{R^2 - \rho^2},$$

and so

$$dl^2 = d\rho^2 + \frac{\rho^2 \, d\rho^2}{R^2 - \rho^2} + \rho^2 \, d\Omega^2,$$

giving

$$dl^2 = \frac{d\rho^2}{1 - (\rho/R)^2} + \rho^2 \, d\Omega^2.$$

This is a homogeneous, isotropic 3D space of (positive) curvature $1/R^2$. Negative and zero curvature are also possible, and setting $\rho = Rr$, all three cases can be expressed as

$$dl^2 = R^2 \left(\frac{dr^2}{1 - kr^2} + r^2 \, d\Omega^2 \right),$$

where $k = -1$, 0 or $+1$.

In general we must allow for R to be an arbitrary function of time $R(t)$ (not position since that would destroy homogeneity), thus we arrive at

$$ds^2 = c^2 \, dt^2 - R^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \right).$$

This is the Friedmann-Robertson-Walker metric. It was first derived by Friedmann in 1922, and then more generally by Robertson and Walker in 1935. It applies to any metric theory of gravity, not just GR.

Deriving

Friedmann-Robertson-Walker

Metric

¿Generality?

here we go

2 Maximally Symmetric Spaces: Uniqueness

We now show that the maximally symmetric spaces are uniquely specified by a "curvature constant" K, and by the numbers of eigenvalues of the metric that are positive or negative. That is, given *two maximally symmetric metrics with the same K and the same numbers of eigenvalues of each sign, it will always be possible to find a coordinate transformation that carries one metric into the other*. Armed with this theorem, we shall be able in the next section to carry out an exhaustive study of maximally symmetric spaces by simply constructing such metrics in one convenient coordinate system.

We showed in the last section that at any given point x in a maximally symmetric space, we can find Killing vectors for which $\xi_\lambda(x)$ vanishes and for which $\xi_{\lambda;\kappa}(x)$ is an arbitrary antisymmetric matrix. It follows then that the co-

Justification of previos step-slide, Weinberg, gravitation & Cosmology

Justification of FRW metric construction, generalization

This is one rather obvious way to carry out this construction. (See Figure 13.1.) Consider a flat $(N+1)$-dimensional space, with metric given by

$$-d\tau^2 = g_{AB}\, dx^A\, dx^B = C_{\mu\nu}\, dx^\mu\, dx^\nu + K^{-1}\, dz^2 \qquad (13.3.1)$$

where $C_{\mu\nu}$ is a constant $N \times N$ matrix and K is some constant. We can embed a non-Euclidean N-dimensional space in this larger space by restricting the variables x^μ and z to the surface of a sphere (or pseudosphere):

$$K C_{\mu\nu} x^\mu x^\nu + z^2 = 1 \qquad (13.3.2)$$

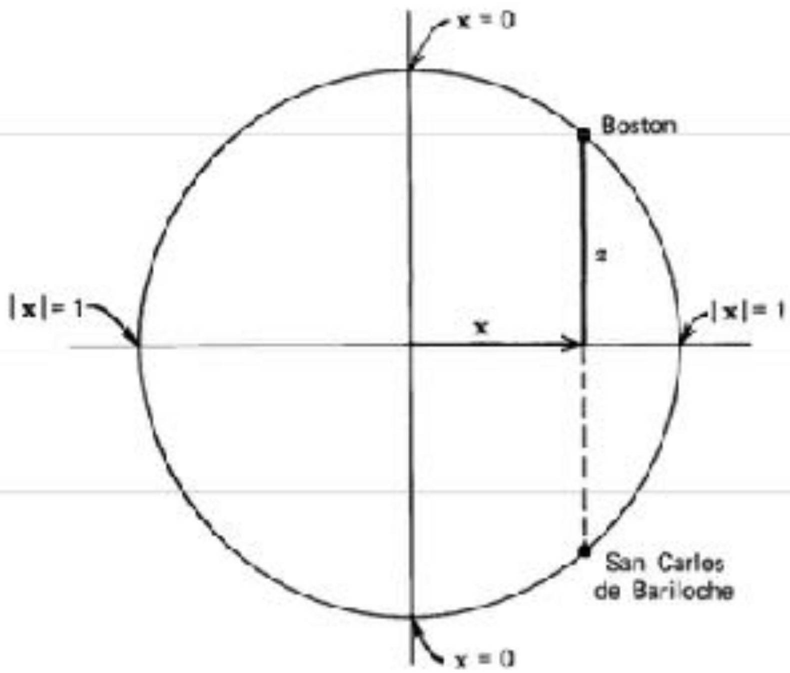


Figure 13.1 Representation of points on a sphere by projection onto the equatorial plane. Note that two points on the sphere correspond to each projected point with given coordinates x^i.

On this surface, dz^2 is given by

$$dz^2 = \frac{K^2 (C_{\mu\nu} x^\mu\, dx^\nu)^2}{z^2} = \frac{K^2 (C_{\mu\nu} x^\mu\, dx^\nu)^2}{(1 - K C_{\rho\sigma} x^\rho x^\sigma)}$$

and therefore (13.3.1) gives

$$-d\tau^2 = C_{\mu\nu}\, dx^\mu\, dx^\nu + \frac{K (C_{\mu\nu} x^\mu\, dx^\nu)^2}{(1 - K C_{\rho\sigma} x^\rho x^\sigma)} \qquad (13.3.3)$$

i.e. spatial terms only. $g_{0i} = 0$ because isotropy $\Longrightarrow$ no preferred direction (cf Schwarzschild). For dl^2 we look for a 3D-space of constant curvature, analagous to the surface of a sphere.

Consider the surface of a sphere in Euclidean 4D. Using Cartesian coordinates (x, y, z, w), but replacing (x, y, z) by spherical polars (ρ, θ, ϕ), we have

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$$\boxed{ds^2 = c^2\, dt^2 - R^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2 \right).}$$

This is the Friedmann-Robertson-Walker metric. It was first derived by Friedmann in 1922, and then more generally by Robertson and Walker in 1935. It applies to any metric theory of gravity, not just GR.

3 Maximally Symmetric Spaces: Construction

Maximally symmetric spaces are essentially unique, so we can learn all about them by constructing examples with arbitrary curvature K in any way we like.

COSMOLOGY

COSMOGRAPHY

The Friedmann Equation

Cosmological Principle ➡ $$ds^2 = c^2 dt^2 - a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

General Relativity ➡ $$G_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

For homogeneous & isotropic Universe: $T^{\mu\nu} = \mathrm{diag}(\rho c^2, -P, -P, -P)$

Substitute FRW metric in $g_{\mu\nu} \longrightarrow \Gamma^{\alpha}_{\beta\gamma} \longrightarrow R^{\alpha}_{\beta\gamma\delta} \longrightarrow \underbrace{R_{\mu\nu} \longrightarrow R}_{G_{\mu\nu}}$

What you get out is:

00- or time-time component: $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{\Lambda c^2}{3}$$

ii- or space-space components: $$\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + 2\frac{Kc^2}{a^2} = 4\pi G\left(\rho - \frac{P}{c^2}\right) + \Lambda c^2$$

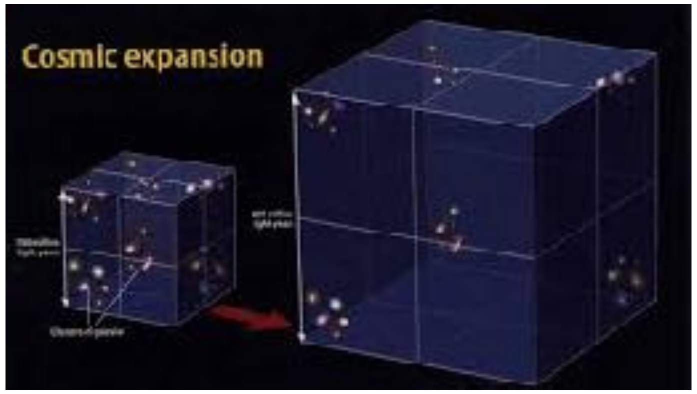
Substituting the 00-component in the ii-component yields

The Friedmann Equation $$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

Students: do this at home; great exercise

Comoving Coordinates

- Remaining degree of freedom (preserving homogeneity and isotropy) is an overall scale factor that relates the geometry (fixed by the radius of curvature R) to physical coordinates – a function of time only



$$d\sigma^2 = a^2(t)d\Sigma^2$$

our conventions are that the scale factor today $a(t_0) \equiv 1$

- Similarly physical distances are given by $d(t) = a(t)D$, $d_A(t) = a(t)D_A$.
- Distances in capital case are *comoving* i.e. they comove with the expansion and do not change with time – simplest coordinates to work out geometrical effects

Hubble Parameter

- Useful to define the expansion rate or Hubble parameter

$$H(t) \equiv \frac{1}{a}\frac{da}{dt}$$ **definition**

since dynamics (Einstein equations) will give this directly as $H(a) \equiv H(t(a))$

- Time becomes

$$t = \int dt = \int \frac{da}{aH(a)}$$ ←→ **the details of cosmology**

- Conformal time becomes

$$\eta = \int \frac{dt}{a} = \int \frac{da}{a^2 H(a)}$$

conformal transformation tries to eliminate expansion effect. Convenient for EMagnetic because conformal transformation preserves Maxwell equation in Minkowsky

Redshift

- Wavelength of light "stretches" with the scale factor, so that it is convenient to define a shift-to-the-red or redshift as the scale factor increases

$$\lambda(a) = a(t)\Lambda$$

$$\frac{\lambda(1)}{\lambda(a)} = \frac{1}{a} \equiv (1+z)$$

$$\frac{\delta\lambda}{\lambda} = -\frac{\delta\nu}{\nu} = z$$

- Given known frequency of emission $\nu(a)$, redshift can be precisely measured (modulo Doppler shifts from peculiar velocities) – interpreting the redshift as a Doppler shift, objects receed in an expanding universe
- Given a measure of distance, $D(z) \equiv D(z(a))$ can be measured

Time and Conformal Time

- Proper time

$$
\begin{aligned}
d\tau^2 &= dt^2 - d\sigma^2 \\
&= dt^2 - a^2(t)d\Sigma^2 \\
&\equiv a^2(t)\left(d\eta^2 - d\Sigma^2\right)
\end{aligned}
$$

- Taking out the scale factor in the time coordinate $d\eta = dt/a$ defines conformal time – useful in that photons travelling radially from observer then obey

$$
\Delta D = \Delta\eta = \int \frac{dt}{a}
$$

so that time and distance may be interchanged

Distance-Redshift Relation

- All distance redshift relations are based on the comoving distance $D(z)$

$$D(a) = \int dD = \int_a^1 \frac{da'}{a'^2 H(a')}$$

$$(da = -(1+z)^{-2} dz = -a^2 dz)$$

$$D(z) = -\int_z^0 \frac{dz'}{H(z')} = \int_0^z \frac{dz'}{H(z')}$$

- Note limiting case is the Hubble law

$$\lim_{z \to 0} D(z) = z/H(z=0) \equiv z/H_0$$

- Hubble constant usually quoted as $H_0 = 100h$ km s^{-1} Mpc^{-1}, observationally $h \sim 0.7$; in natural units $H_0 = (2997.9)^{-1} h$ Mpc^{-1} defines an inverse length scale

Horizon

- Distance travelled by a photon in the whole lifetime of the universe defines the horizon
- By $d\tau = 0$, the horizon is simply the conformal time elapsed

$$D_{\text{horizon}}(t) = \int_0^t \frac{dt'}{a} = \eta(t)$$

- Since the horizon always grows with time, there is always a point in time before which two observers separated by a distance D could not have been in causal contact
- Horizon problem: why is the universe homogeneous and isotropic on large scales, near the current horizon – problem deepens for objects seen at early times, e.g. CMB

Evolution of Scale Factor

- FRW cosmology is fully specified if the function $a(t)$ is given
- General relativity relates the scale factor with the matter content of universe.
- Build the Einstein tensor $G_{\mu\nu}$ out of the metric and use Einstein equation

$$G_{\mu\nu} = -8\pi G T_{\mu\nu}$$

$$G^0{}_0 = -\frac{3}{a^2}\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{R^2}\right]$$

$$G^i{}_j = -\frac{1}{a^2}\left[2\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{R^2}\right]\delta^i{}_j$$

Einstein Equations

- Isotropy demands that the stress-energy tensor take the form

$$T^0_{\ 0} = \rho$$
$$T^i_{\ j} = -p\delta^i_{\ j}$$

where ρ is the energy density and p is the pressure

- So Einstein equations become

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{R^2} = \frac{8\pi G}{3} a^2 \rho$$
$$2\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{R^2} = -8\pi G a^2 p$$
$$\text{or} \quad \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 = -\frac{4\pi G}{3} a^2 (\rho + 3p)$$

Friedman Equations

- More usual to see Einstein equations expressed in time not conformal time

$$\frac{\dot{a}}{a} = \frac{da}{d\eta}\frac{1}{a} = \frac{da}{dt} = aH(a)$$

$$\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 = \frac{d}{d\eta}\left(\frac{\dot{a}}{a}\right) = a\frac{d}{dt}\left(\frac{da}{dt}\right) = a\frac{d^2a}{dt^2}$$

- Friedmann equations:

$$H^2(a) + \frac{1}{a^2R^2} = \frac{8\pi G}{3}\rho$$

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}(\rho + 3p)$$

- Convenient fiction to describe curvature as an energy density component $\rho_K = -3/(8\pi G a^2 R^2) \propto a^{-2}$

Critical Density

- Friedmann equation for H then reads

$$H^2(a) = \frac{8\pi G}{3}(\rho + \rho_K) \equiv \frac{8\pi G}{3}\rho_c$$

defining a critical density today ρ_c in terms of the expansion rate

- In particular, its value today is given by the Hubble constant as

$$\rho_{\rm c}(z=0) = 3H_0^2/8\pi G = 1.8788 \times 10^{-29} h^2 {\rm g\,cm}^{-3}$$

- Energy density today is given as a fraction of critical $\Omega \equiv \rho/\rho_c|_{z=0}$. Radius of curvature then given by $R^{-2} = H_0^2(\Omega - 1)$
- If $\Omega \approx 1, \rho \approx \rho_c$, then $\rho_K \ll \rho_c$ or $H_0 R \ll 1$, universe is flat across the Hubble distance. $\Omega < 1$ negatively curved; $\Omega > 1$ positively curved

Conservation Law

- Second Friedmann equation, or acceleration equation, simply expresses energy conservation (why: stress energy is automatically conserved in GR via Bianchi identity)

$$d\rho V + p dV = 0$$

$$d\rho a^3 + p da^3 = 0$$

$$\dot{\rho} a^3 + 3\frac{\dot{a}}{a}\rho a^3 + 3\frac{\dot{a}}{a} p a^3 = 0$$

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a} \qquad w \equiv p/\rho$$

- If $w =$ const. then the energy density depends on the scale factor as $\rho \propto a^{-3(1+w)}$.

Acceleration Equation

- Time derivative of (first) Friedman equation

$$
\begin{aligned}
2\frac{1}{a}\frac{da}{dt}\left[\frac{1}{a}\frac{d^2a}{dt^2} - H^2(a)\right] &= \frac{8\pi G}{3}\frac{d\rho_c}{dt} \\
\left[\frac{1}{a}\frac{d^2a}{dt^2} - \frac{8\pi G}{3}\rho_c\right] &= \frac{4\pi G}{3}[-3(1+w_c)\rho_c] \\
\frac{1}{a}\frac{d^2a}{dt^2} &= -\frac{4\pi G}{3}[(1+3w_c)\rho_c] \\
&= -\frac{4\pi G}{3}(\rho + \rho_K + 3p + 3p_K) \\
&= -\frac{4\pi G}{3}(1+3w)\rho
\end{aligned}
$$

- Acceleration equation says that universe decelerates if $w > -1/3$

Multicomponent Universe

- The total energy density can be composed of a sum of components with differing equations of state

$$\rho(a) = \sum_i \rho_i(a) = \sum_i \rho_i(a=1) a^{-3(1+w_i)}, \quad \Omega_i \equiv \rho_i/\rho_c|_{a=1}$$

- Important cases: nonrelativistic matter $\rho_m = m n_m \propto a^{-3}$, $w_m = 0$; relativistic radiation $\rho_r = E n_r = \nu n_r \propto a^{-4}, w_r = 1/3$; "curvature" $\rho_K \propto a^{-2}, w_K = -1/3$; constant energy density or cosmological constant $\rho_\Lambda \propto a^0, w_\Lambda = -1$
- Or generally with $w_c = p_c/\rho_c = (p + p_K)/(\rho + \rho_K)$

$$\rho_c(a) = \rho_c(a=1) e^{-\int d\ln a \, 3(1+w_c(a))}$$

$$H^2(a) = H_0^2 e^{-\int d\ln a \, 3(1+w_c(a))}$$

Expansion Required

- Friedmann equations "predict" the expansion of the universe. Non-expanding conditions $da/dt = 0$ and $d^2a/dt^2 = 0$ require

$$\rho = -\rho_K \qquad \rho = -3p$$

i.e. a positive curvature and a total equation of state $w \equiv p/\rho = -1/3$

- Since matter is known to exist, one can in principle achieve this with

$$\rho = \rho_m + \rho_\Lambda = -\rho_K = -3p = 3\rho_\Lambda$$

$$\rho_\Lambda = -\frac{1}{3}\rho_K \quad \rho_m = -\frac{2}{3}\rho_K$$

Einstein introduced ρ_Λ for exactly this reason – "biggest blunder"; but note that this balance is unstable: ρ_m can be perturbed but ρ_Λ, a true constant cannot

Expressing Distances in an Expanding Universe

The geometry and expansion rate of the Universe effects angular sizes and distances measured. Integrate over components of RW metric.

defines causality

$D_H = c/H_o$ → Hubble Distance (distance light travels in Hubble time, $t_H = 1/H_o$)

$D_C = D_H \int_0^z \frac{dz}{E(z)}$. → Radial Co-moving Distance

$D_M = D_C$ (flat) → Transverse Co-moving Distance, differs for curved space (see Hogg 2000)

$IntrinsicSize = d\theta x D_M(z)$

$D_A(z) = D_M(z)/(1+z)$

D_A = L(proper length)/θ(angular size) = $D_M/(1+z)$ → Angular Distance

D_L = sqrt (L/4π*flux) = $D_M(1+z) = D_A(1+z)^2$ → Luminosity Distance

Homework generate plots of DL,DA vs z for different cosmologies

If $\Lambda = 0$ and flat geometry, then

$D_L = 2c/H_o\ [z/(G+1)]\ \{1+[z/(G+1)]\}$ where $G = (1 + z)^{1/2}$

See Ned Wright's Javascript Cosmology Calculator for D_L for different cosmologies:

http://www.astro.ucla.edu/~wright/CosmoCalc.html

E(z)=\sqrt{\Omega_m(1+z)^3+\Omega_k(1+z)^2+\Omega_\Lambda}

Different Time/z Dependence for Different Cosmologies. Can we use it?

Angular diameter distance vs z
(plotting D_A/D_H where $D_A = L/\theta$)

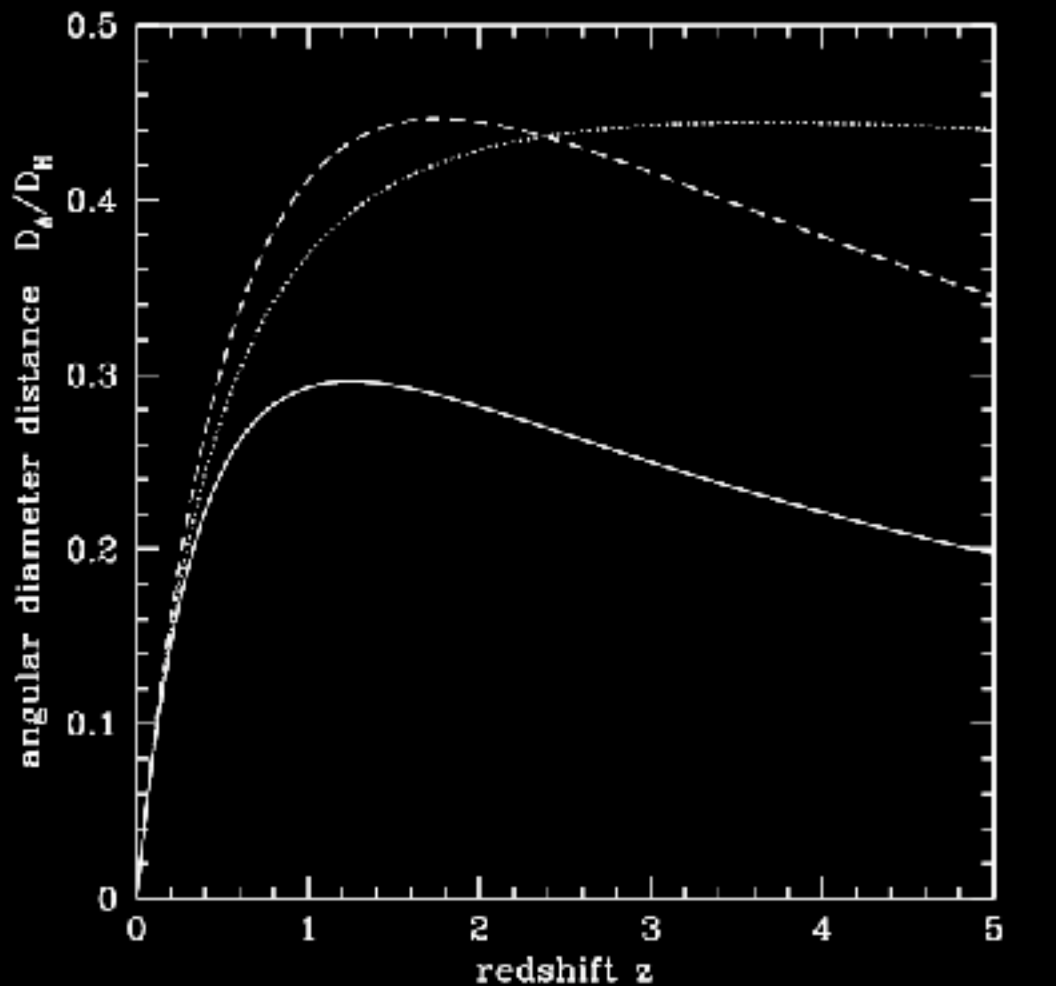


Figure 2: The dimensionless angular diameter distance D_A/D_H. The three curves are for the three world models, $(\Omega_M, \Omega_\Lambda) = (1, 0)$, solid; (0.05, 0), dotted; and (0.2, 0.8), dashed.

Luminosity distance vs z
(plotting D_L/D_H)

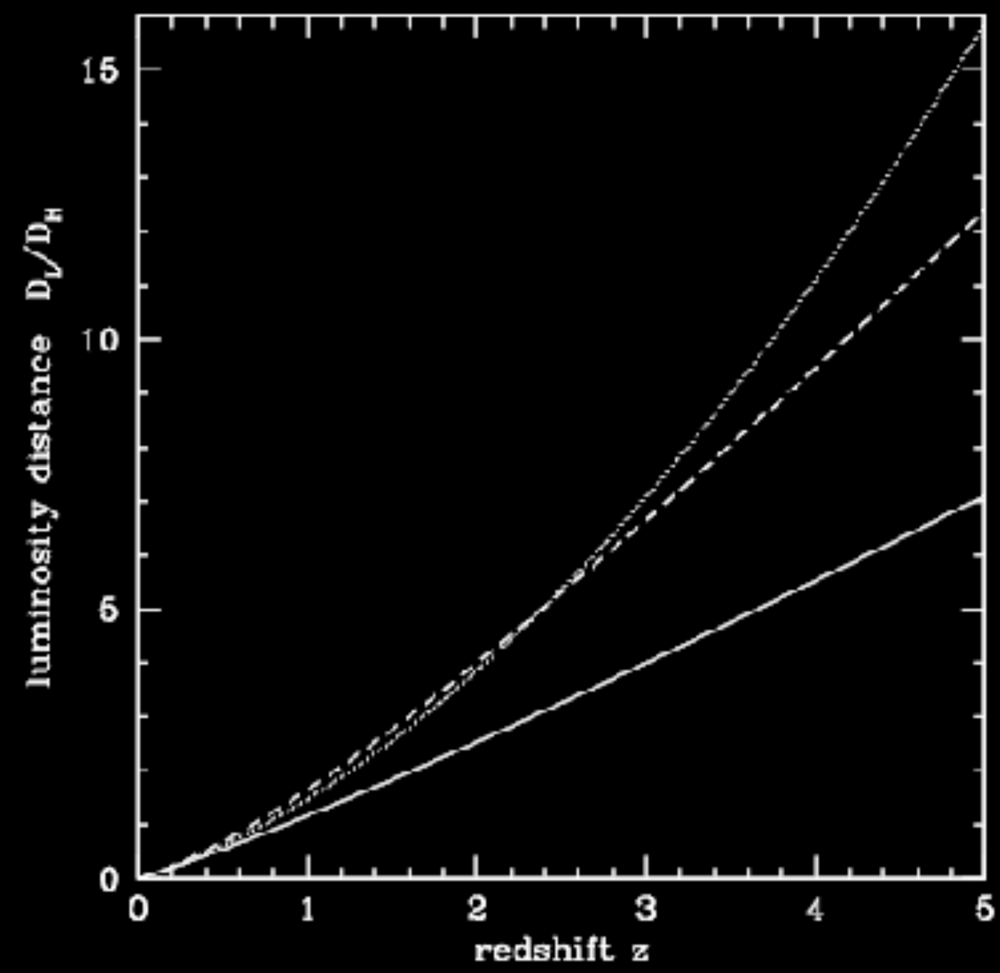


Figure 3: The dimensionless luminosity distance D_L/D_H. The three curves are for the three world models, $(\Omega_M, \Omega_\Lambda) = (1, 0)$, solid; (0.05, 0), dotted; and (0.2, 0.8), dashed.

$D_H = c/H_o = 3000 h^{-1}$ Mpc

At high z, angular diameter distance is such that 1 arcsec is about 5 kpc.

flat, Λ=0 – solid
open, Λ=0 – dotted
flat, non-zero Λ - dashed

(from Hogg 2000 astro-ph 9905116)