MHD Waves & Shocks

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MHD Waves

In compressible fluids:

✓**acoustic (sound) waves**

In plasmas:

✓Besides these: **new modes appear**

✓ To study these modes, we will consider for simplicity the MHD equations in **IDEAL form**

IDEAL MHD Equations

CGS

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0\,,$$ *(continuity)*

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) + \nabla p - \frac{1}{c}\vec{J}\times\vec{B} = 0\,,$$ *(momentum)*

$$\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla p + \gamma p \nabla\cdot\mathbf{v} = 0\,,$$ *(internal energy)*

$v_e = v$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla}\times(\vec{v}_e \times \vec{B})$$ ***(magnetic induction)***

$$\vec{J} = \frac{c}{4\pi}\vec{\nabla}\times\vec{B} \qquad \nabla\cdot\mathbf{B} = 0\,.$$ *(Ampere, divergencia)*

Eq. of state to close the system:

$$p = nk_B T$$ (if ideal gas)

MHD Equations (ideal)

$$\frac{\partial \vec{B}}{\partial t} + (\vec{v}.\vec{\nabla})\vec{B} + \vec{B}(\vec{\nabla}.\vec{v}) - (\vec{B}.\vec{\nabla})\vec{v} = 0$$

$$\frac{\partial \rho}{\partial t} + \vec{v}.\vec{\nabla}\rho + \rho\vec{\nabla}.\vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v}.\vec{\nabla}\vec{v} + \frac{k_B}{\overline{m}\rho}\vec{\nabla}(\rho T) - \frac{1}{4\pi\rho}(\vec{\nabla} \times \vec{B}) \times \vec{B} = 0$$

$$\frac{k_B}{\overline{m}}\left(\frac{1}{\gamma - 1}\rho\frac{\partial T}{\partial t} + \frac{1}{\gamma - 1}\rho\vec{v} \cdot \vec{\nabla}T - T\frac{\partial \rho}{\partial t} - T\vec{v} \cdot \vec{\nabla}\rho\right) = 0$$

MHD Equations (ideal)

$\vec{\nabla} \times (\vec{v}_e \times \vec{B})$

$$\frac{\partial \vec{B}}{\partial t} + (\vec{v}.\vec{\nabla})\vec{B} + \vec{B}(\vec{\nabla}.\vec{v}) - (\vec{B}.\vec{\nabla})\vec{v} = 0$$

$$\frac{\partial \rho}{\partial t} + \vec{v}.\vec{\nabla}\rho + \rho\vec{\nabla}.\vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v}.\vec{\nabla}\vec{v} + \frac{k_B}{m\rho}\vec{\nabla}(\rho T) - \frac{1}{4\pi\rho}(\vec{\nabla} \times \vec{B}) \times \vec{B} = 0$$

$$\frac{k_B}{m}\left(\frac{1}{\gamma - 1}\rho\frac{\partial T}{\partial t} + \frac{1}{\gamma - 1}\rho\vec{v} \cdot \vec{\nabla}T - T\frac{\partial \rho}{\partial t} - T\vec{v} \cdot \vec{\nabla}\rho\right) = 0$$

MHD Equations (ideal)

$$\frac{\partial \vec{B}}{\partial t} + (\vec{v}.\vec{\nabla})\vec{B} + \vec{B}(\vec{\nabla}.\vec{v}) - (\vec{B}.\vec{\nabla})\vec{v} = 0$$

$$\frac{\partial \rho}{\partial t} + \vec{v}.\vec{\nabla}\rho + \rho\vec{\nabla}.\vec{v} = 0$$

$$p = \rho k_B T / \overline{m}$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v}.\vec{\nabla}\vec{v} + \frac{k_B}{\overline{m}\rho}\vec{\nabla}(\rho T) - \frac{1}{4\pi\rho}(\vec{\nabla} \times \vec{B}) \times \vec{B} = 0$$

$$\frac{k_B}{\overline{m}}\left(\frac{1}{\gamma - 1}\rho\frac{\partial T}{\partial t} + \frac{1}{\gamma - 1}\rho\vec{v} \cdot \vec{\nabla}T - T\frac{\partial \rho}{\partial t} - T\vec{v} \cdot \vec{\nabla}\rho\right) = 0$$

MHD Waves

Hypotheses:

- **B_o, ρ_o, v_o, T_o** : constant and uniform equilibrium

- Neglecting radiative cooling, heating -> **adiabatic flow**

- **Let us perturb** this system: assuming small perturbations

$$f(\vec{x}, \psi) = f_o + f_1(\vec{x}, t) + f_2(\vec{x}, t) + \ldots$$

Physical quantity at equilibrium (0-order)   1st –order perturbation: $f_1 << f_o$

- **Neglect terms of higher order:** $f_1^2, f_2, g_2^2, g_2, f_1 g_1,$ etc.

- $\boldsymbol{v = v_o + v_1}$, *take* $\boldsymbol{v_o = 0}$ -> $\boldsymbol{v_1 = v}$

Substituting *f* in MHD eqs.

$$f(\vec{x}, \psi) = f_o + f_1(\vec{x}, t) + f_2(\vec{x}, t) + \ldots$$

Equations of 1st order

$$\frac{\partial \vec{B}_1}{\partial t} - (\vec{B}_o.\vec{\nabla})\vec{v} + \vec{B}_o(\vec{\nabla}.\vec{v}) = 0$$

$$\frac{\partial \rho_1}{\partial t} + \rho_o \vec{\nabla}.\vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \frac{k_B}{\overline{m}}\vec{\nabla}T_1 + \frac{kT_o}{\overline{m}\rho_o}\vec{\nabla}\rho_1 + \frac{1}{4\pi\rho_o}\vec{B}_o \times (\vec{\nabla} \times \vec{B}_1) = 0$$

$$\frac{k_B}{\overline{m}}\left(\frac{\rho_o}{\gamma - 1}\frac{\partial T_1}{\partial t} - T_o\frac{\partial \rho_1}{\partial t}\right) = 0$$

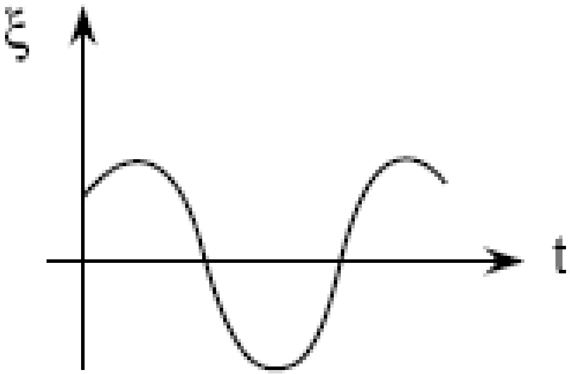
1st order equations solutions

- Have constant coefficients:

→ solutions:

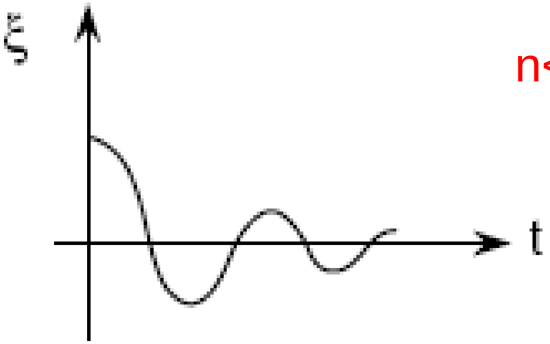
$$f_1(\vec{x},t) = f_1 \exp(nt + i\vec{k}\cdot\vec{x})$$

- ✓ If n imaginary (or complex) and k real -> ***WAVE***
- ✓ If n real and n>0 -> **INSTABILITY** : ***f ~ exp (nt)*** , n growth rate

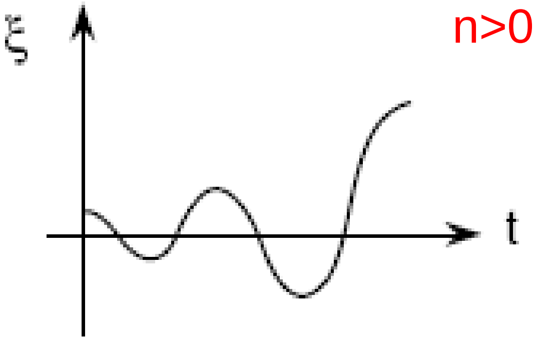


n imaginary

Wave



n<0



n>0

Damping wave

Instability

1st order diff. eqs. -> algebraic

Substituting: $f_1(\vec{x},t) = f_1 \exp(nt + i\vec{k}\cdot\vec{x})$

$$n\vec{B}_1 - i(\vec{k}.\vec{B}_o)\vec{v} + i\vec{B}_o(\vec{k}.\vec{v}) = 0$$

$$n\frac{\rho_1}{\rho_o} + i\vec{k}.\vec{v} = 0$$

$$\frac{T_1}{T_o} = \frac{\rho_1}{\rho_o} \rightarrow \frac{p_1}{p_o} = \frac{\rho_1}{\rho_o} + \frac{T_1}{T_o} = \gamma\frac{\rho_1}{\rho_o}$$

$$n\vec{v} + i\gamma\frac{p_o}{\rho_o} i\vec{k}\frac{\rho_1}{\rho_o} + \frac{i}{4\pi\rho_o}\vec{B}_o \times (\vec{k}\times\vec{B}_1) = 0$$

1st order equation

- Subst. B_1 and ρ_1 into v eq.:

$$n\vec{v} + \frac{\gamma \quad v_s^2}{n}\vec{k}(\vec{k}\cdot\vec{v}) - \frac{1}{4\pi\rho_o}\vec{B}_o \times \left[(\vec{k}\times\vec{v})(\vec{k}\cdot\vec{B}_o) - (\vec{k}\cdot\vec{v})(\vec{k}\times\vec{B}_o)\right] = 0$$

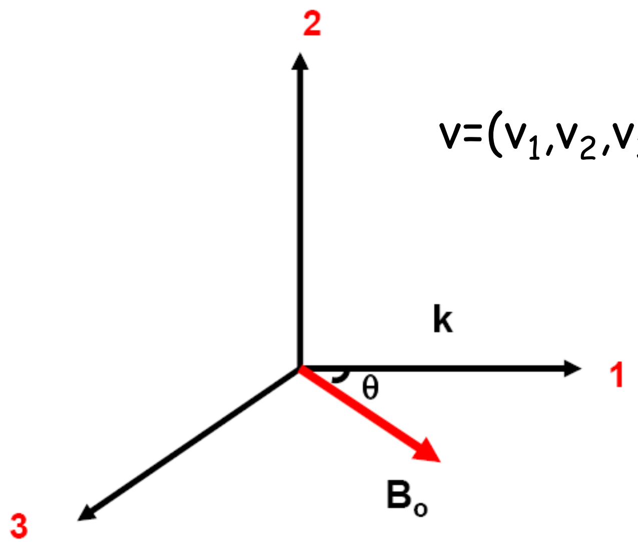
$$v_s = \left(\frac{p_o}{\rho_o}\right)^{1/2} = \left(\frac{k_B T_o}{m}\right)^{1/2}$$

isothermal

sound speed

Want dispersion relation *n(k)*

Let us consider the following system of coordinates (1,2,3):



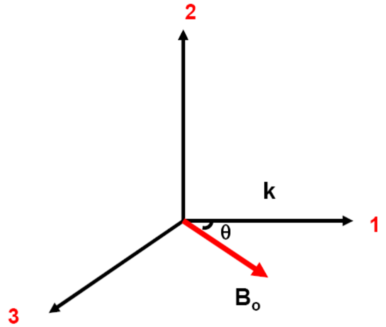
$v=(v_1,v_2,v_3)$

Let us introduce the Alfven speed:

$$v_A = \frac{B_o}{(4\pi\rho_o)^{1/2}}$$

Substituting into 1st order eq.

v_1 $$nv_1 + \frac{\gamma \alpha v_s^2 k^2}{n} v_1 + \frac{k^2 v_A^2}{n} \sin\theta(-\cos\theta v_3 + \sin\theta v_1) = 0$$



v_2 $$nv_2 + \frac{k^2 v_A^2}{n} \cos^2\theta v_2 = 0$$

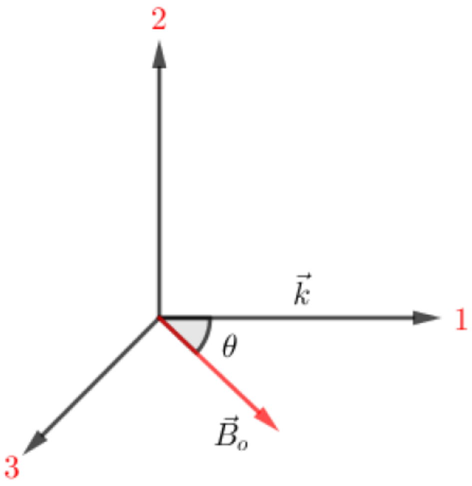
v_3 $$nv_3 + \frac{k^2 v_A^2}{n} \cos\theta(\cos\theta v_3 - \sin\theta v_1) = 0$$

These homogeneous eqs. -> null determinant of coefficients: gives the dispersion relation n(k)

Alfven Waves

- **One of the solutions of the 1st order eqs. (v_|_ k):**

$$n = \pm i k v_A \cos\theta$$

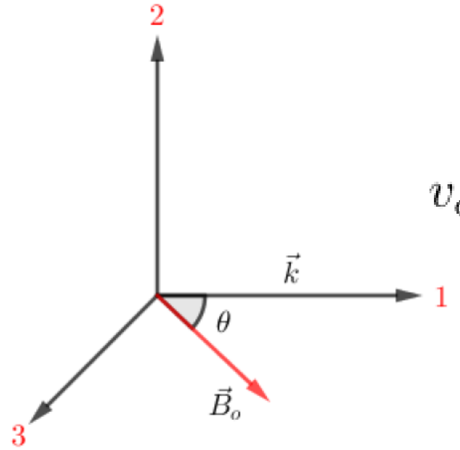


Which corresponds to waves with phase velocity:

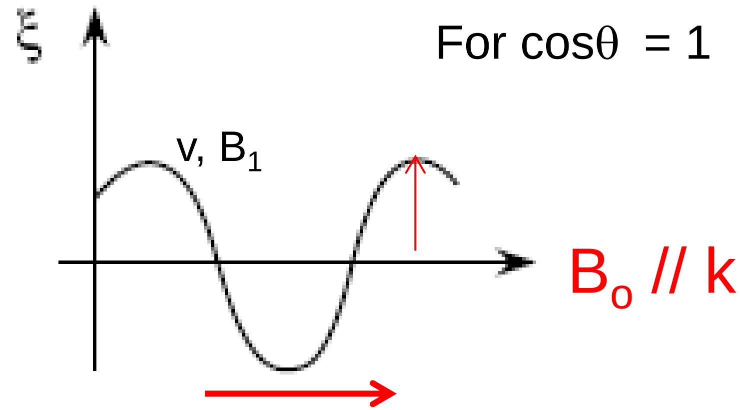
$$v_\phi = \left(\frac{\partial x_1}{\partial t}\right)_\phi = \mp v_A \cos\theta = \mp \frac{B_o}{\sqrt{4\pi\rho_o}} \cos\theta$$

These waves : first described by Alfven in 1953 -> **Alfven wave**

Alfven Wave Nature



$$v_\phi = \frac{B_o}{\sqrt{4\pi\rho_o}}\cos\theta$$



$$v_\phi = \frac{B_0}{\sqrt{4\pi\rho}}$$

- ✓ Waves (disturbances): **v perpendicular (transverse)** to **k and B_o**
- ✓ Propagate // B_o
- ✓ Flow curved by the motion of the wave and a restoring force is exerted by the lines due to the **magnetic tension** **(~ to wave in a string with tension):**

v_ϕ^2 = (tension/density)

$$\frac{B_0^2}{4\pi\rho}$$

Alfven Waves

- ✓ They are directly **observed in laboratory, Earth's magnetosphere and in solar wind**
- ✓ Have important role in astrophysical plasmas in general
- ✓ Key role in the transmission of forces (like acoustic waves in non-magnetized gas)

Ex: magnetized fluid approaching an obstacle (a cloud):

- **if $v < v_A$** : A-waves *warn* fluid about obstacle
- **if $v > v_A$** : there is no time for the waves to *warn*: a shock forms (see later)

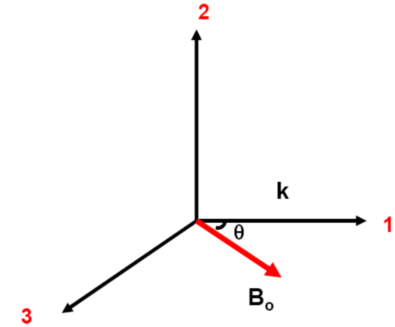
More about Alfven Waves

- Note that the magnetic pressure can be written as:

$$\frac{B^2}{8\pi} = \frac{1}{2}\rho v_A^2$$

(Leave as an exercise for you to demonstrate it!)

Going back to 1st order eqs.



v_1

$$n v_1 + \frac{\gamma \alpha v_s^2 k^2}{n} v_1 + \frac{k^2 v_A^2}{n} \sin\theta(-\cos\theta v_3 + \sin\theta v_1) = 0$$

v_2

$$n v_2 + \frac{k^2 v_A^2}{n} \cos^2\theta v_2 \quad = 0$$

v_3

$$n v_3 + \frac{k^2 v_A^2}{n} \cos\theta(\cos\theta v_3 - \sin\theta v_1) \quad = 0$$

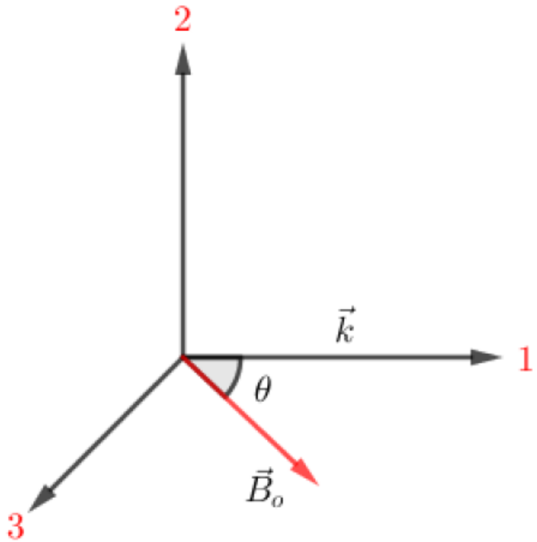
These homogeneous eqs. -> null determinant of coefficients: gives the dispersion relation n(k)

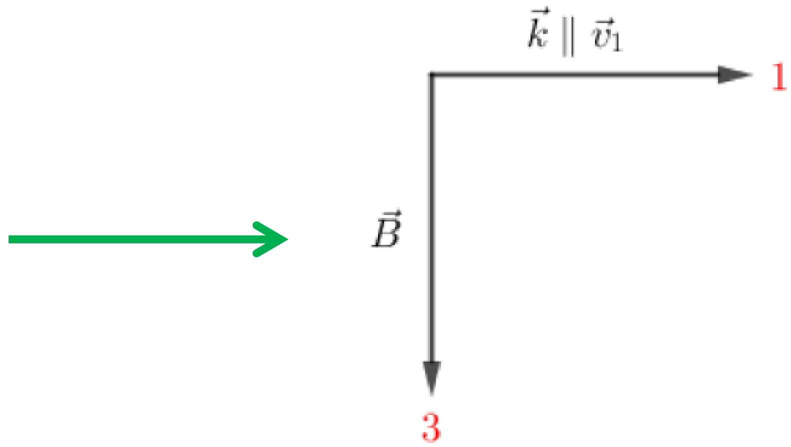
Magneto-acoustic waves

- Let us now consider **for simplicity :**

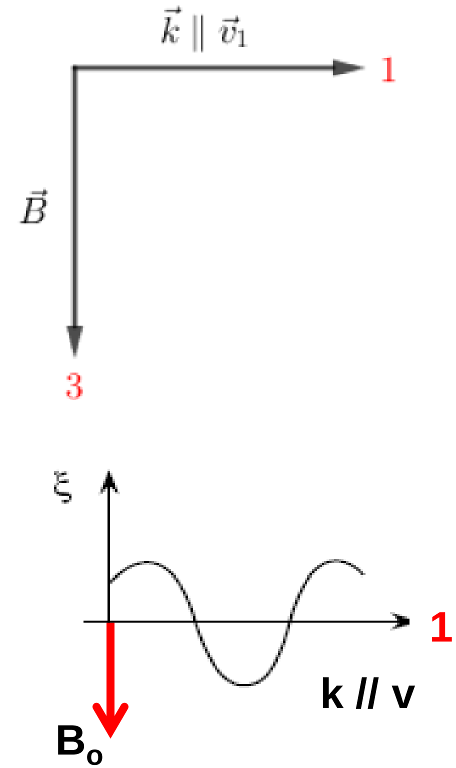
v // to 1 -> v // k

- And: $$\theta = \pi/2\ (\vec{k} \perp \vec{B}_o)$$





Magneto-acoustic waves



- In this case we have **longitudinal waves (v // k)**
- with phase velocity :

$$v_\phi = \pm(v_A^2 + \gamma v_S^2)^{1/2}$$

- If we write the total pressure:

$$p_{tot} = \frac{B^2}{8\pi} + p$$

- **Then:**

Adiabatic: $p \propto \rho^\gamma$

$$\left(\frac{\partial p_{tot}}{\partial \rho}\right)_S = \gamma v_S^2 + \frac{B_o^2}{4\pi\rho_o} = \gamma v_S^2 + v_A^2 \rightarrow v_\phi = \left(\frac{\partial p_{tot}}{\partial \rho}\right)_S^{1/2}$$

magnetoacoustic wave is like an acoustic wave in which the compressive movements encounter resistance not only from ***thermal pressure*** but also from ***magnetic pressure*** (with compression perp. to the magnetic lines)

Magneto-acoustic waves

$$v_s = \left(\frac{p_o}{\rho_o}\right)^{1/2} = \left(\frac{k_B T_o}{m}\right)^{1/2}$$

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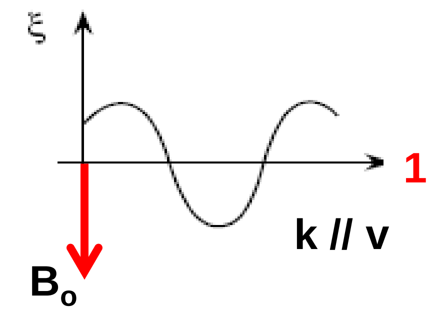
$$v_\phi = \pm(v_A^2 + \gamma v_S^2)^{1/2}$$



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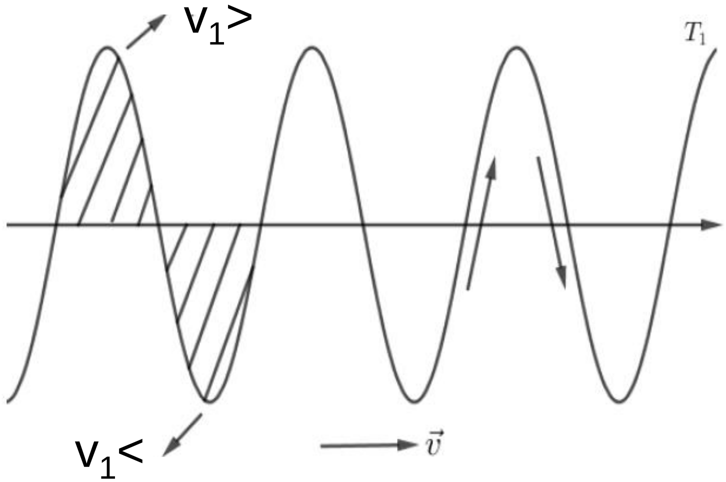
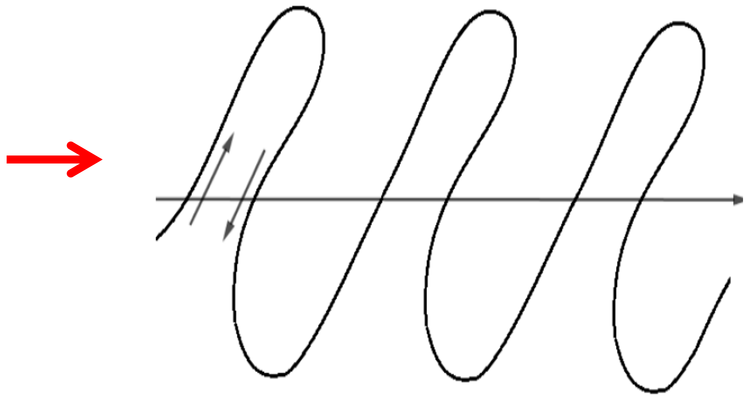
magnetoacoustic wave is like an acoustic wave in which the compressive movements encounter resistance not only from ***thermal pressure*** but also from ***magnetic pressure*** (with compression perp. to the magnetic lines)

Shock Waves

Shock waves

- When the "perturbation" velocity $\mathbf{v_1}$ becomes too large the wave **"breaks"** (like sea waves when arriving at the beach):





- In a **hydrodynamic** fluid this happens when the **velocity > sound speed**:

$$M = \frac{v_1}{\gamma^{1/2} v_S} > 1$$

-> At the position where this happens: we have a ***SHOCK DISCONTINUITY***

Shock Wave: concept

- Mechanism of acoustic wave "break“ -> collisional interaction of fast particles behind-the-wave with slow particles just ahead
- This process occurs when the two sets of particles are separated from each other only by few mean-free-paths (~mfp)
- **Δx = ~ mfp** -> **shock thickness** (discontinuity)
- Fluid theory can tell us WHERE the wave breaks, where Δx, but cannot describe inside **Δx**
- **We can use MHD conservation equations to describe conditions behind and ahead the SHOCK discontinuity**
- The only parameter: is the strength of the **shock or its velocity**:

-> **MACH number**:

$$M = \frac{v_1}{\gamma^{1/2} v_S}$$

MHD Equations (ideal)

$$\frac{\partial \vec{B}}{\partial t} + (\vec{v}.\vec{\nabla})\vec{B} + \vec{B}(\vec{\nabla}.\vec{v}) - (\vec{B}.\vec{\nabla})\vec{v} = 0$$

$$\frac{\partial \rho}{\partial t} + \vec{v}.\vec{\nabla}\rho + \rho\vec{\nabla}.\vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v}.\vec{\nabla}\vec{v} + \frac{k_B}{\overline{m}\rho}\vec{\nabla}(\rho T) - \frac{1}{4\pi\rho}(\vec{\nabla} \times \vec{B}) \times \vec{B} = 0$$

$$\frac{k_B}{\overline{m}}\left(\frac{1}{\gamma - 1}\rho\frac{\partial T}{\partial t} + \frac{1}{\gamma - 1}\rho\vec{v} \cdot \vec{\nabla}T - T\frac{\partial \rho}{\partial t} - T\vec{v} \cdot \vec{\nabla}\rho\right) = 0$$

But let us write ideal MHD equations in **conservative form**

- **Conservative form** is usually given by:

$$\frac{\partial}{\partial t}(\text{stuff}) + \nabla \cdot (\text{flux of stuff}) = 0$$

where source and sink terms go on the RHS

  - If 'stuff' is a scalar, then 'flux of stuff' is a vector
  - If 'stuff' is a vector, then 'flux of stuff' is a dyadic tensor
- If this equation is satisfied, then 'stuff' is locally conserved

We have seen continuity equation in conservative form

- The continuity equation is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

Where **V** is velocity here

where $\rho \mathbf{V}$ is the mass flux

- Mass is locally conserved

Momentum equation in conservative form

- The momentum equation can be written as

$$\frac{\partial \rho \mathbf{V}}{\partial t} + \nabla \cdot \mathbf{T} = 0$$

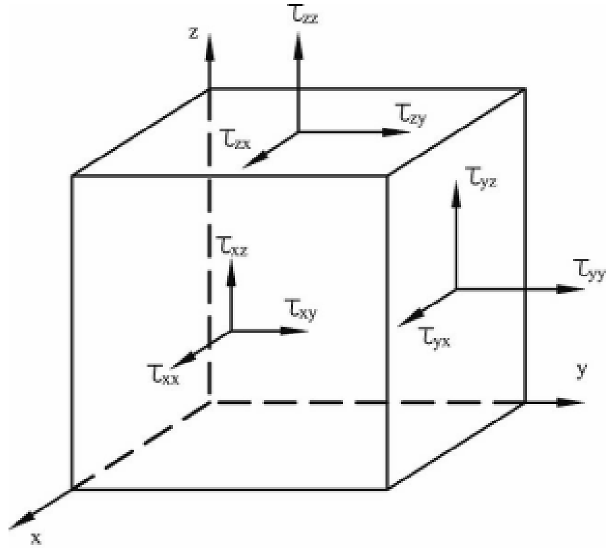
where the ideal MHD stress tensor $\mathbf{T}$ is given by

$$\mathbf{T} = \rho \mathbf{V}\mathbf{V} + \left(p + \frac{B^2}{8\pi} \right) \mathbf{I} - \frac{\mathbf{B}\mathbf{B}}{4\pi}$$

and $\mathbf{I} \equiv \hat{\mathbf{e}}_1 \hat{\mathbf{e}}_1 + \hat{\mathbf{e}}_2 \hat{\mathbf{e}}_2 + \hat{\mathbf{e}}_3 \hat{\mathbf{e}}_3$ is the identity dyadic tensor

- The quantity $p + B^2/8\pi$ is called the total pressure

Physical meaning of stress tensor



(N. Murphy's courtesy)

- Imagine you have an infinitesimal box
- Forces are exerted on each face by the outside volume
- The forces on each side of the box could have components in three directions
- The *stress tensor* includes the nine quantities needed to describe the total force exerted on this box by the outside

Reynolds Stress Tensor

- The Reynolds stress is given by

$$\mathbf{T}_V \equiv \rho \mathbf{V}\mathbf{V}$$

- $\mathbf{T}_V$ represents the flux of momentum. Think:

$$\mathbf{T}_V \equiv \underbrace{\rho \mathbf{V}}_{\text{Momentum density}} \underbrace{\mathbf{V}}_{\text{times a velocity}}$$

- $(\rho V_x) V_y$ is the rate at which the x component of momentum is carried in the y direction (and vice versa)

Maxwell Stress Tensor

- The Maxwell stress is

$$\mathbf{T}_B \equiv \frac{B^2}{8\pi}\mathbf{I} - \frac{\mathbf{BB}}{4\pi}$$

- The quantity $\frac{\mathbf{BB}}{4\pi}$ is called the *hoop stress*
- Key point: Momentum is transported by the magnetic field

….The divergence of this Tensor gives the Lorenz Force

$$\nabla \cdot \mathbf{T}_B = J \times B \quad !!$$

Conservative mass, momentum and energy equations

- Conservation of mass, momentum, & energy are given by

$$\begin{aligned}\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{V}) &= 0\\ \frac{\partial\rho\mathbf{V}}{\partial t} + \nabla\cdot\mathbf{T} &= 0\\ \frac{\partial w}{\partial t} + \nabla\cdot\mathbf{s} &= 0\end{aligned}$$

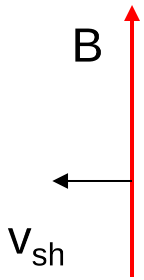
where the stress tensor, energy density, and energy flux are

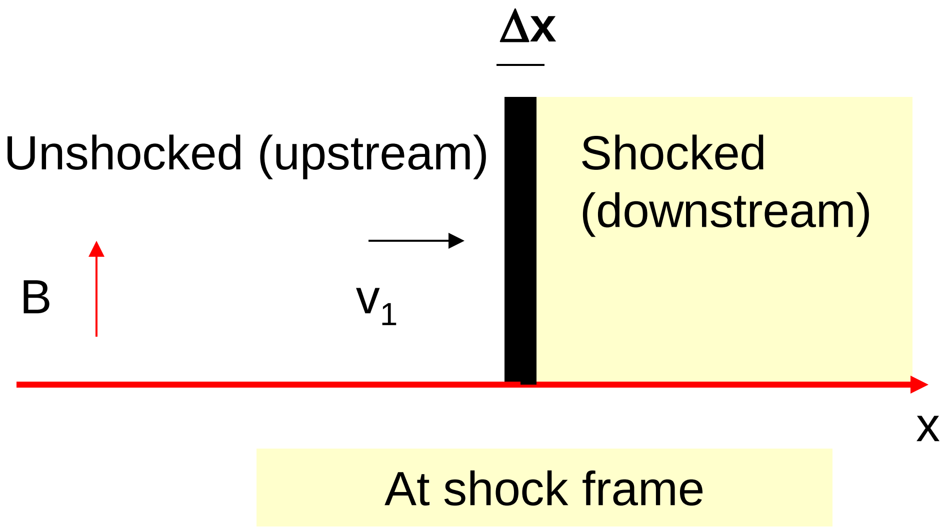
$$\begin{aligned}\mathbf{T} &= \rho\mathbf{V}\mathbf{V} + \left(p + \frac{B^2}{8\pi}\right)\mathbf{I} - \frac{\mathbf{B}\mathbf{B}}{4\pi}\\ w &= \frac{\rho V^2}{2} + \frac{B^2}{8\pi} + \frac{p}{\gamma-1}\\ \mathbf{s} &= \left(\frac{\rho V^2}{2} + \frac{\gamma}{\gamma-1}p\right)\mathbf{V} + \frac{\mathbf{E}\times\mathbf{B}}{4\pi}\end{aligned}$$

$\mathbf{E} + \frac{\mathbf{V}\times\mathbf{B}}{c} = 0$ in ideal MHD (Ohm's law)

Studying Shocks

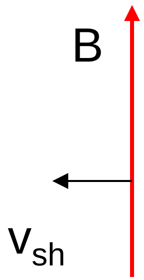
- Let us take a shock discontinuity
- Adopt a coordinate system at rest in the shock front
- Let us assume stationary flux ($\partial / \partial t = 0$)
- Plasma before shock (upstream, unshocked gas) moves with v_1 with respect to shock front
- Shock direction along –x
- B $\perp$ to shock direction: y

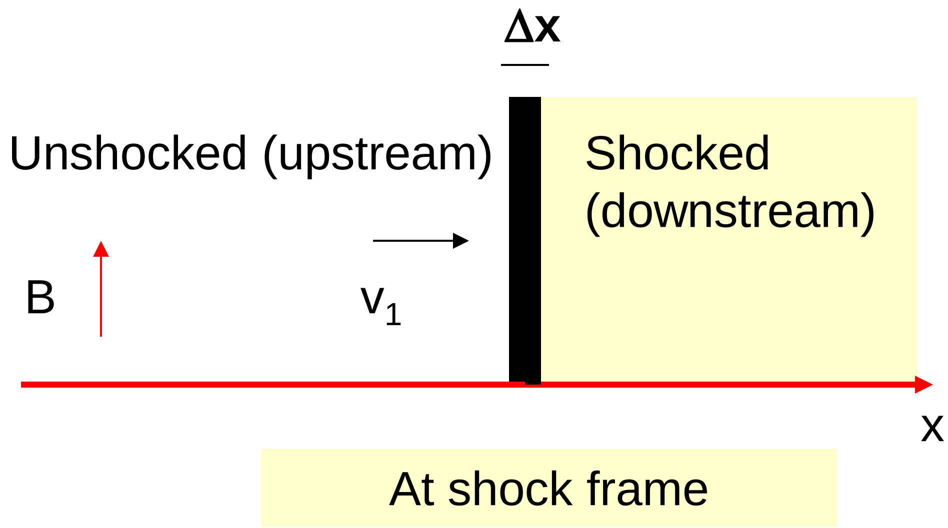




Note that:

The case of propagation // to B can be obtained by making **B = 0** in the equations, since // motion to B does not affect and is not affected by B and so does not exchange energy with the field





Ideal MHD eqs in conservative form

- We integrate in volume the MHD eqs. in conservative form

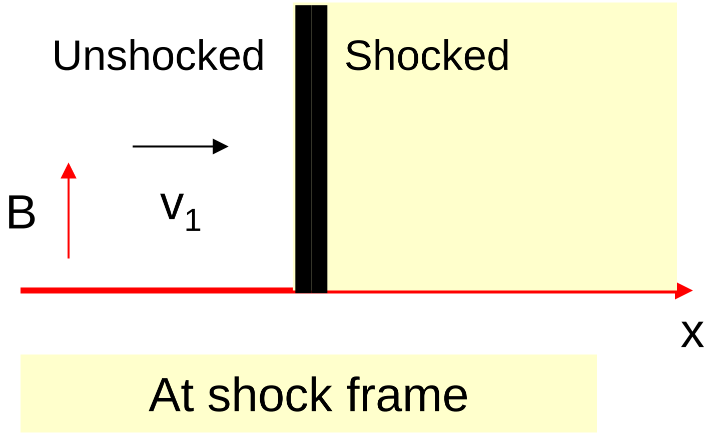
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

$$\frac{\partial \rho \mathbf{V}}{\partial t} + \nabla \cdot \mathbf{T} = 0$$

$$\frac{\partial w}{\partial t} + \nabla \cdot \mathbf{s} = 0$$

- And : $\frac{\partial \vec{B}}{\partial t} = -(\vec{v} \cdot \vec{\nabla})\vec{B} + (\vec{B} \cdot \vec{\nabla})\vec{v} - \vec{B}(\vec{\nabla} \cdot \vec{v}) = 0$

- Assuming $\partial / \partial t = 0$



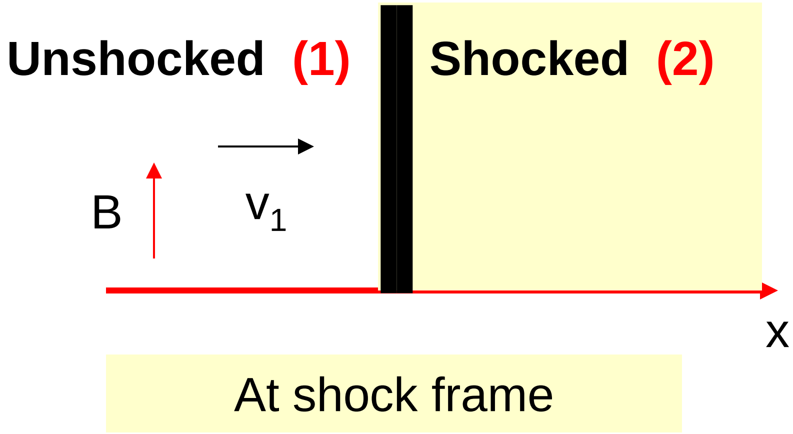
Rankine-Hugoniot Relations

$$[\rho v] = 0$$

$$\left[\frac{B}{\rho}\right] = 0$$

$$\left[\rho v^2 + \frac{B^2}{8\pi} + \frac{\rho k_B T}{\overline{m}}\right] = 0$$

$$\left[\frac{1}{2}v^2 + \frac{B^2}{4\pi\rho} + \frac{\gamma}{\gamma - 1}\frac{k_B T}{\overline{m}}\right] = 0$$



- Where:

$$[\rho v] = 0$$

$$\rho_1 v_1 - \rho_2 v_2 = 0$$

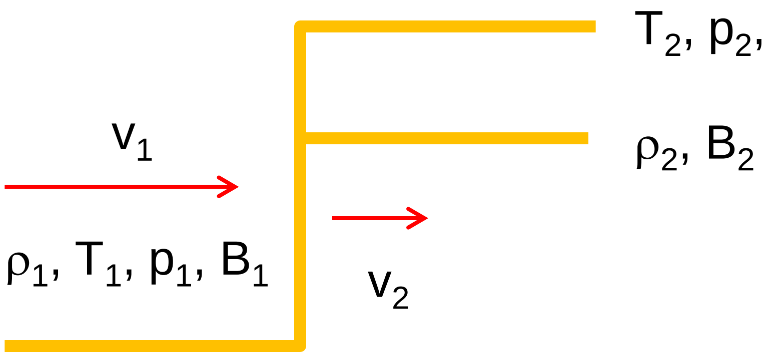
Rankine-Hugoniot Relations

$$[\rho v] = 0$$

$$\left[\frac{B}{\rho}\right] = 0$$

$$\left[\rho v^2 + \frac{B^2}{8\pi} + \frac{\rho k_B T}{\overline{m}}\right] = 0$$

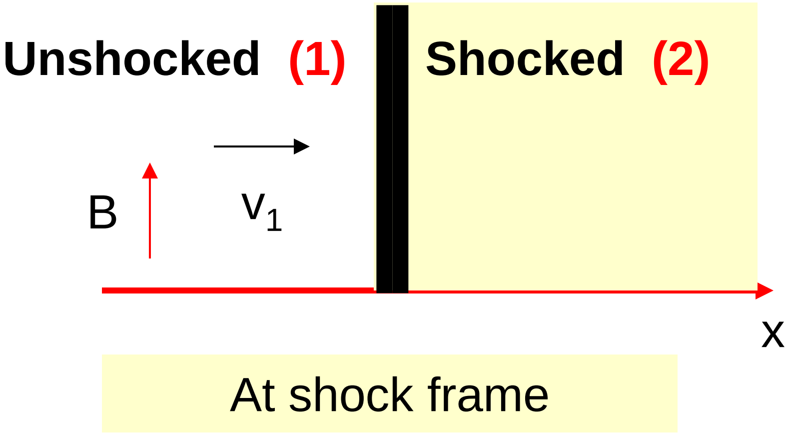
$$\left[\frac{1}{2}v^2 + \frac{B^2}{4\pi\rho} + \frac{\gamma}{\gamma - 1}\frac{k_B T}{\overline{m}}\right] = 0$$



- Where:

$$[\rho v] = 0$$

$$\rho_1 v_1 - \rho_2 v_2 = 0$$



Rankine-Hugoniot Relations

- Parameters:

$$M = \frac{v_1}{\left(\frac{\gamma k_B T_1}{m}\right)^{1/2}} = \frac{v_1}{\gamma^{1/2} v_{S1}}$$ Mach number

$$x = \frac{p_2}{p_1} = \frac{\rho_2 T_2}{\rho_1 T_1} :$$ Pressure ratio

$$y = \frac{\rho_2}{\rho_1} :$$ Density ratio

- y:

$$2(2-\gamma)\beta y^2 + \gamma\left[(\gamma-1)M^2 + 2(1+\beta)\right]y - \gamma(\gamma+1)M^2 = 0$$

- x:

$$x = \frac{y\frac{(\gamma+1)}{(\gamma-1)} - 1 + \beta(y-1)^3}{\frac{\gamma+1}{\gamma-1} - y}$$

- Here: $\beta = \frac{B_1^2}{8\pi p_1}$

For shock with B=0

- Or B // v:

$$y = \frac{(\gamma+1)M^2}{(\gamma-1)M^2+2} \qquad x = \frac{2\gamma M^2 - (\gamma-1)}{\gamma+1}$$

- For M=1 -> y = x =1 -> **WEAK SHOCK**
- For M>>1:

$$\frac{\rho_2}{\rho_1} = y \rightarrow \frac{\gamma+1}{\gamma-1} \left(= 4 \text{ para } \gamma = 5/3\right)$$

$$\frac{p_2}{p_1} = x \rightarrow \frac{2\gamma}{\gamma+1}M^2 \left(= \frac{5}{4}M^2 \text{ para } \gamma = \frac{5}{3}\right)$$

$$\frac{T_2}{T_1} = \frac{x}{y} \rightarrow \frac{2\gamma(\gamma-1)}{(\gamma+1)^2}M^2 \left(= \frac{5}{16}M^2 \text{ para } \gamma = \frac{5}{3}\right)$$

-> STRONG SHOCK!

For a shock with B≠0

$$2(2-\gamma)\beta y^2 + \gamma\left[(\gamma-1)M^2 + 2(1+\beta)\right]y - \gamma(\gamma+1)M^2 = 0$$

$$x = \frac{y^{\frac{(\gamma+1)}{(\gamma-1)}} - 1 + \beta(y-1)^3}{\frac{\gamma+1}{\gamma-1} - y} \qquad y = \frac{\rho_2}{\rho_1} \qquad x = \frac{p_2}{p_1}$$

For a WEAK SHOCK -> let us make: **y=1**

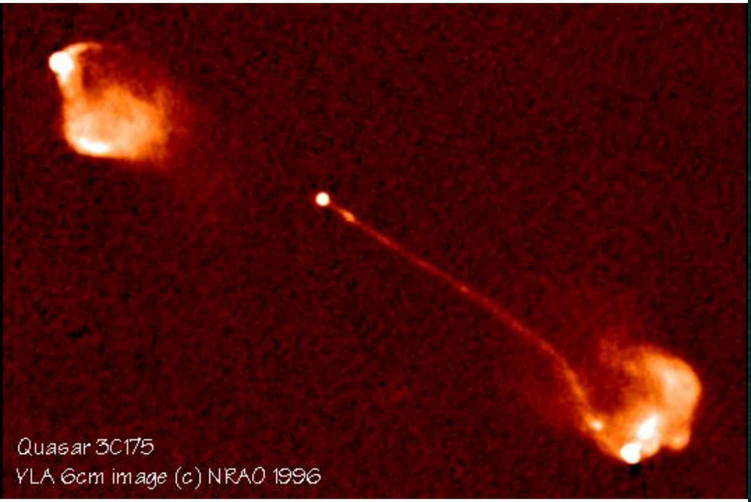
We find:

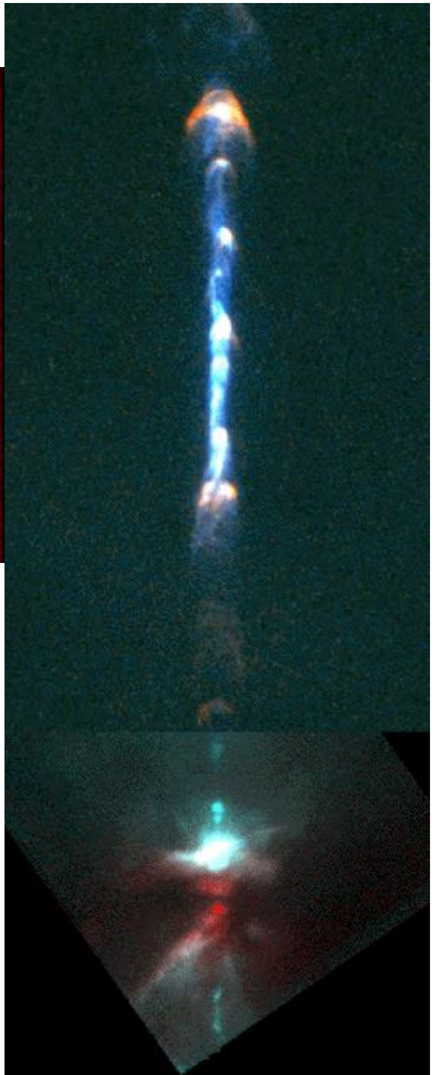
$$\gamma M^2 = \gamma + 2\beta \quad \Rightarrow \quad v_1^2 = \gamma v_S^2 + v_A^2$$

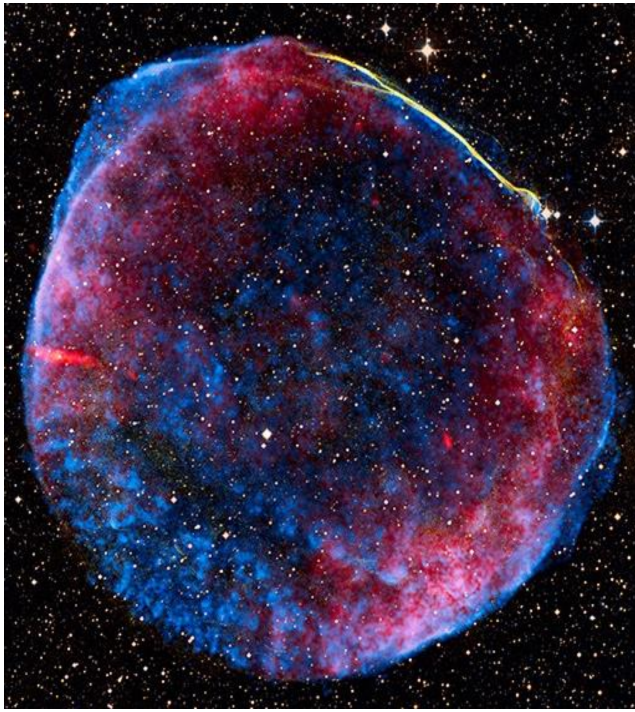
Therefore, for $\vec{B} \neq 0$ **there is SHOCK only if:** $v_1^2 > \gamma v_s^2 + v_A^2$

-> Only if the fluid velocity is larger than the magnetosonic speed!!

Exs. of Astrophysical Shocks







Bow shocks at jets as in this active galaxy above and this protostar at right

Supernova Remnants: Shocks due to the explosion of massive stars

End of Class 3