MHD for ASTROPHYSICISTS

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NOTES:http://www.astro.iag.usp.br/~dalpino/?q=teaching
(references therein)

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Plasmas in Astrophysics

Class 1

Contents

- What is plasma?
- Why plasmas in astrophysics?
- Quasi-neutrality

What is PLASMA?

- Gas with sufficient number of **free charged particles** (positive + negative) → so that its behaviour dominated by **electromagnetic forces**.

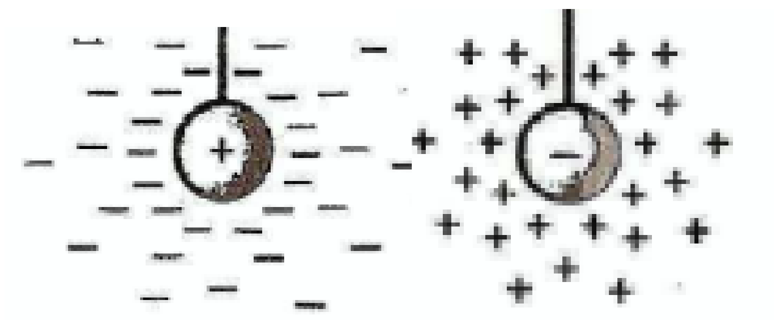
- Even **low ionization** degree (**n_{ch}/n_{total}~1%**) : sufficient for gas to show electromagnetic properties (electrical conductivity ~ fully ionized gas) -> **PLASMA!**

More formal PLASMA Concept

Plasma: **quasi-neutral gas of charged particles** (electrons+ions):

$$\mathbf{e\ n_e \sim e\ Z\ n_i}$$

with long range ***collective interactions***:



(*binary collisions not important*)

Why Plasmas in Astrophysics ?

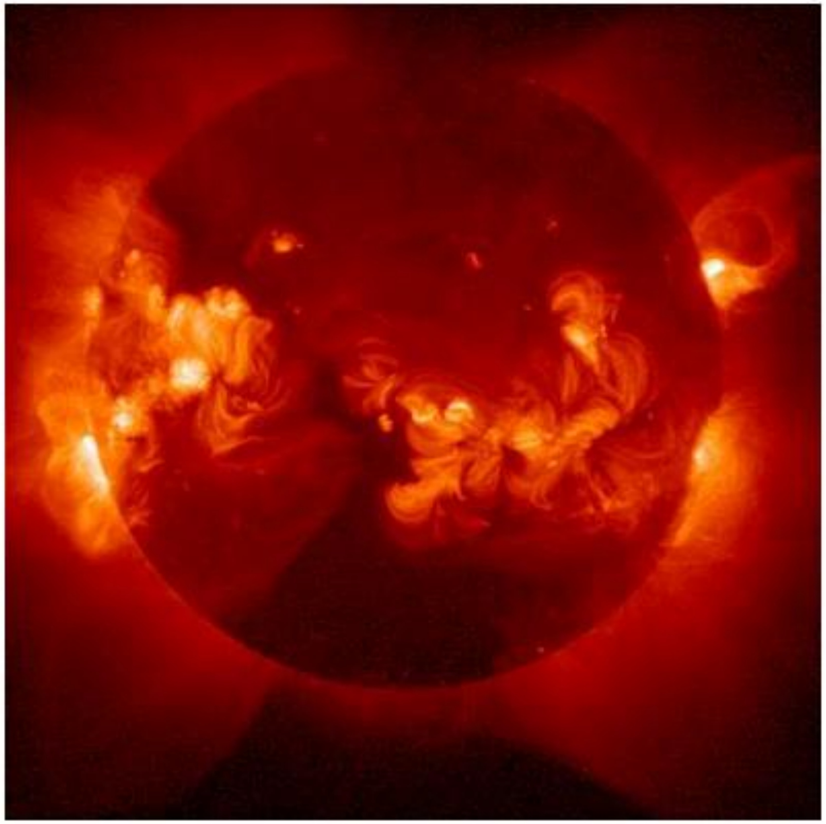
The universe does not consist of ordinary matter

Over **90% of visible matter in Universe**:

PLASMA !

Exs. Astrophysical Plasmas

Example: The Sun

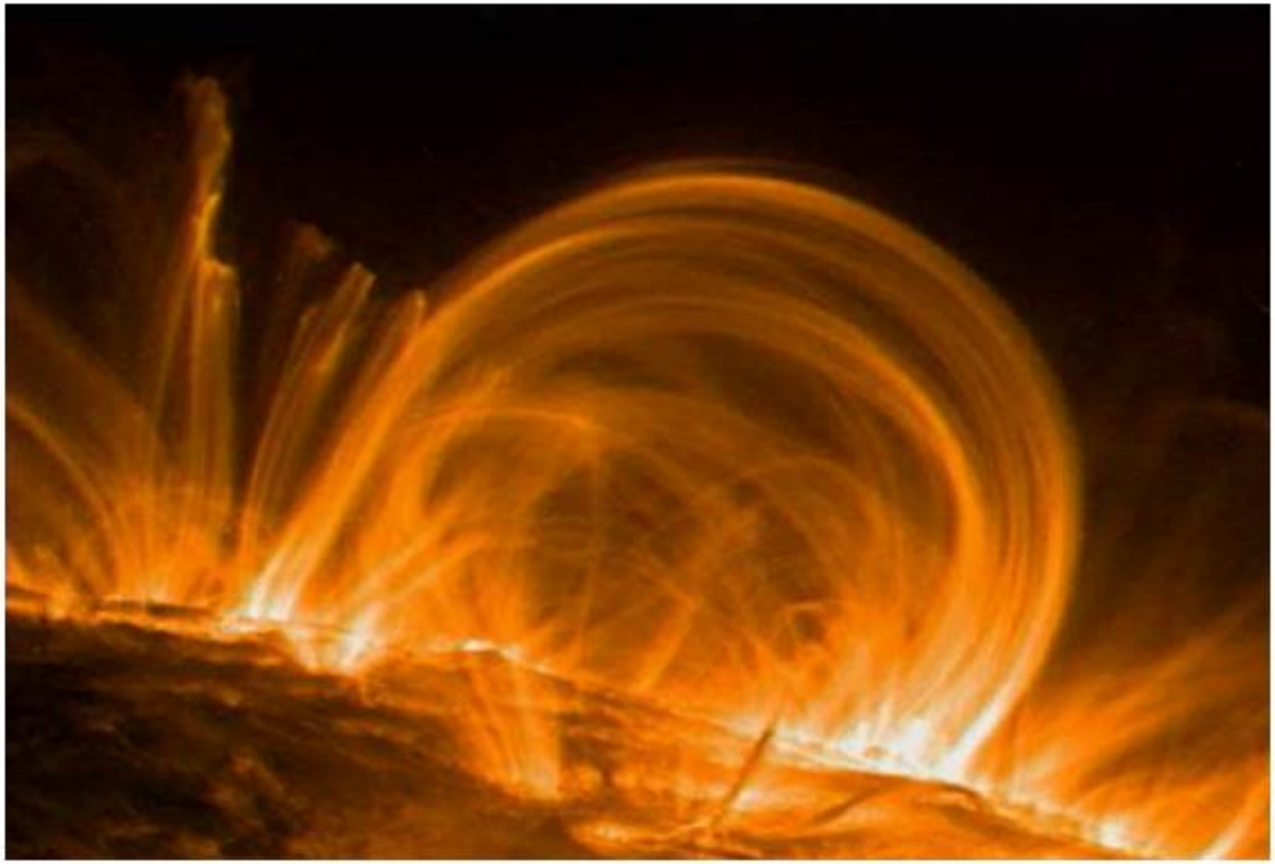


a magnetized plasma!

(sunatallwavelengths.mpeg)

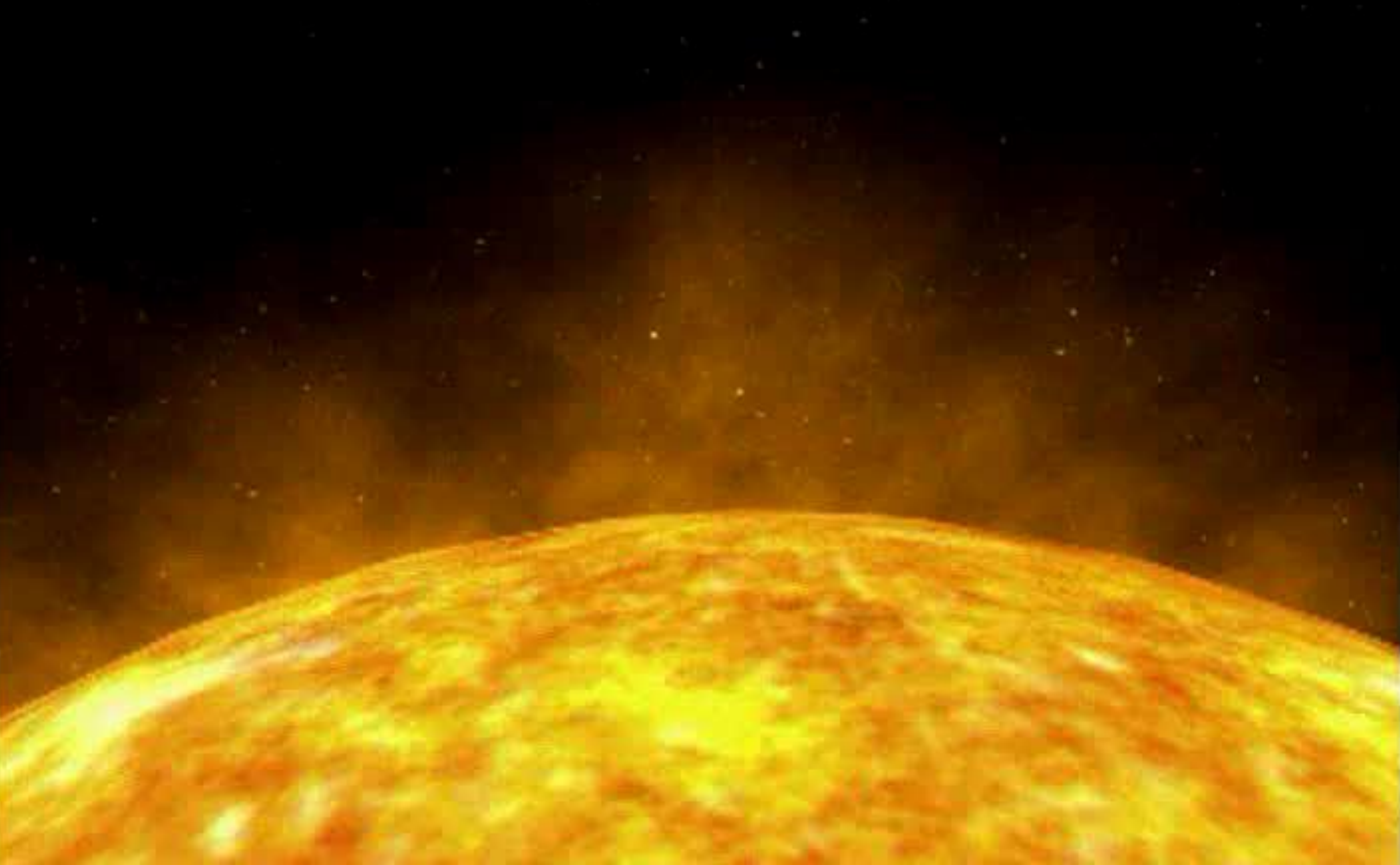
Exs. Astrophysical Plasmas

Example: Coronal loops (cont'd)

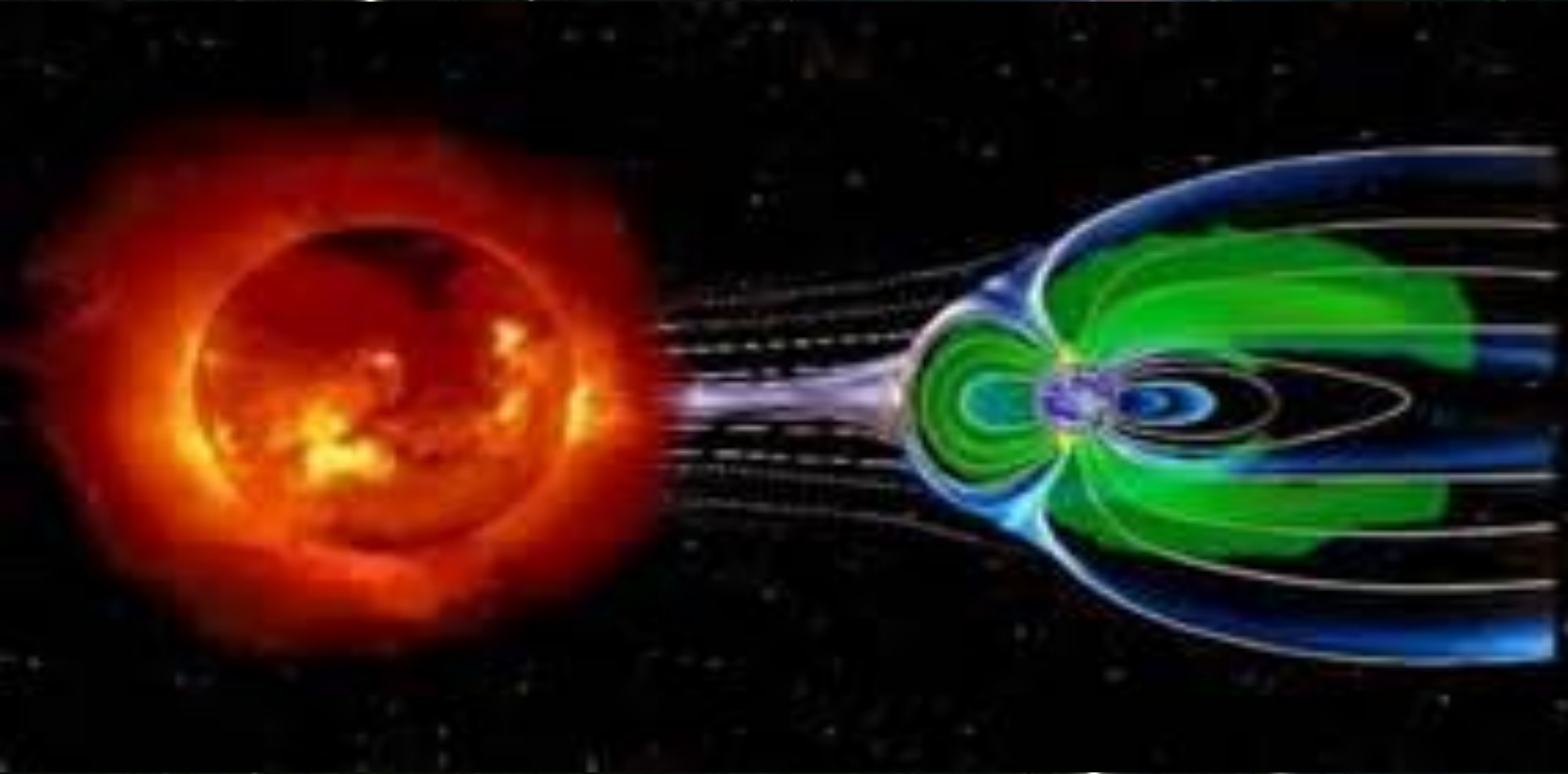


[from recent observations with TRACE spacecraft]

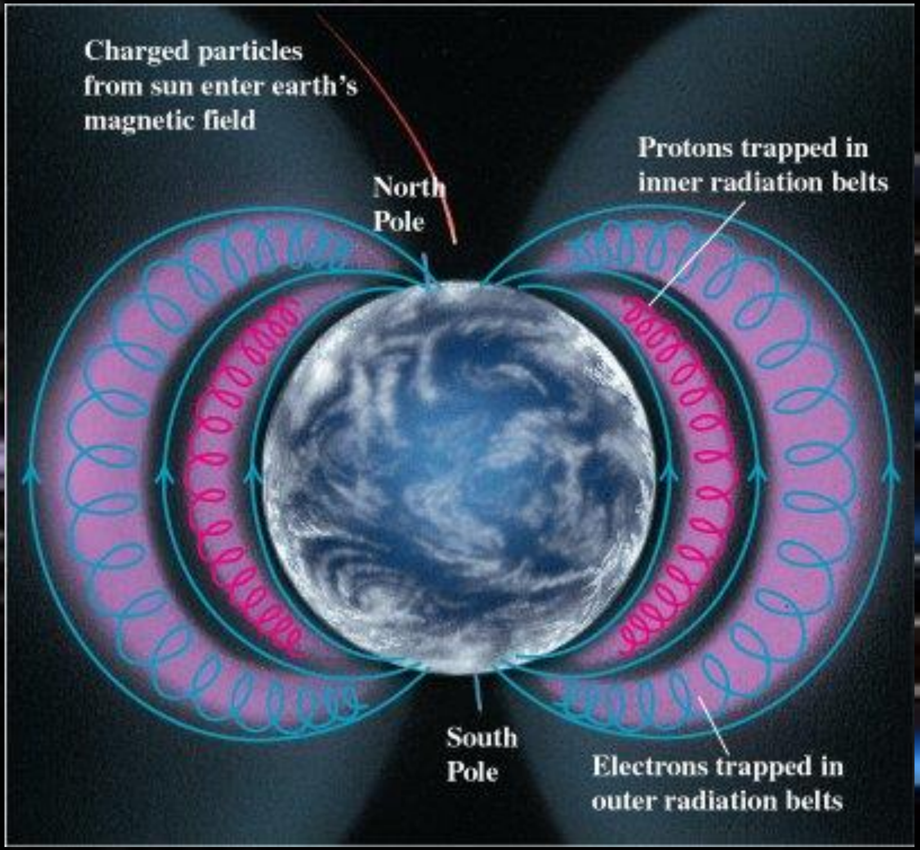
Sun and Earth



Ex. Plasmas in Earth



Ex. Plasmas in Earth



Importance of Plasmas

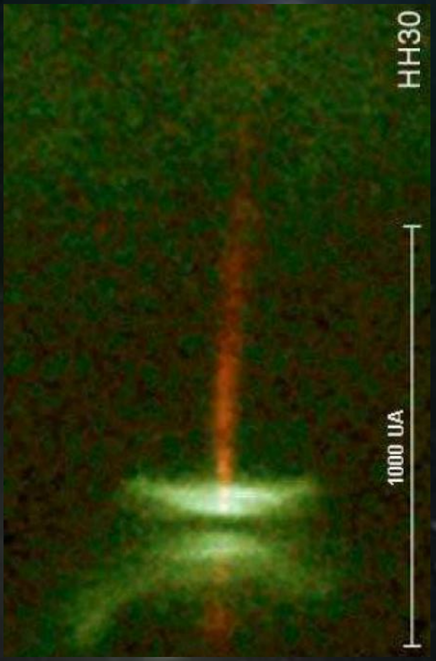
- **Magnetized plasmas are present in almost all astrophysical objects**

- They are ***crucial*** in:

- star formation; late stages

- solar and stellar activity

- formation of jets and accretion disks



- formation and propagation of cosmic rays

- galaxy structure

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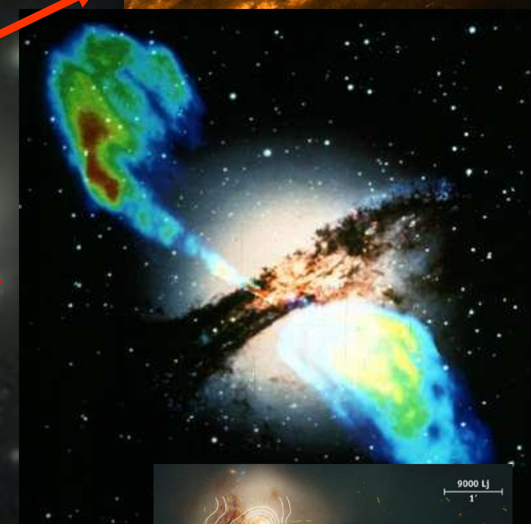
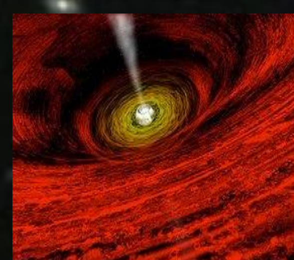
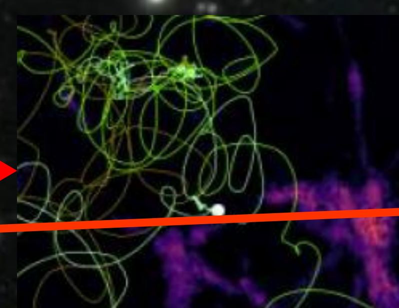





Importance of Plasmas

- They are also ***crucial*** in:

  - ISM

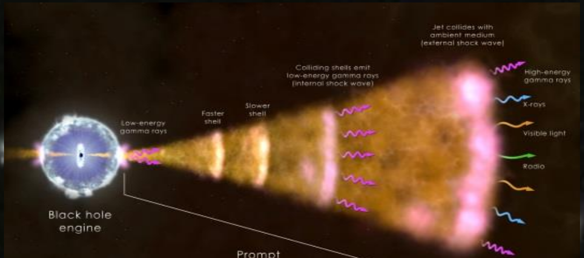
  - molecular clouds

  - supernova remnants

  - proto-planetary disks

  - planetary nebulae

  - GRBs



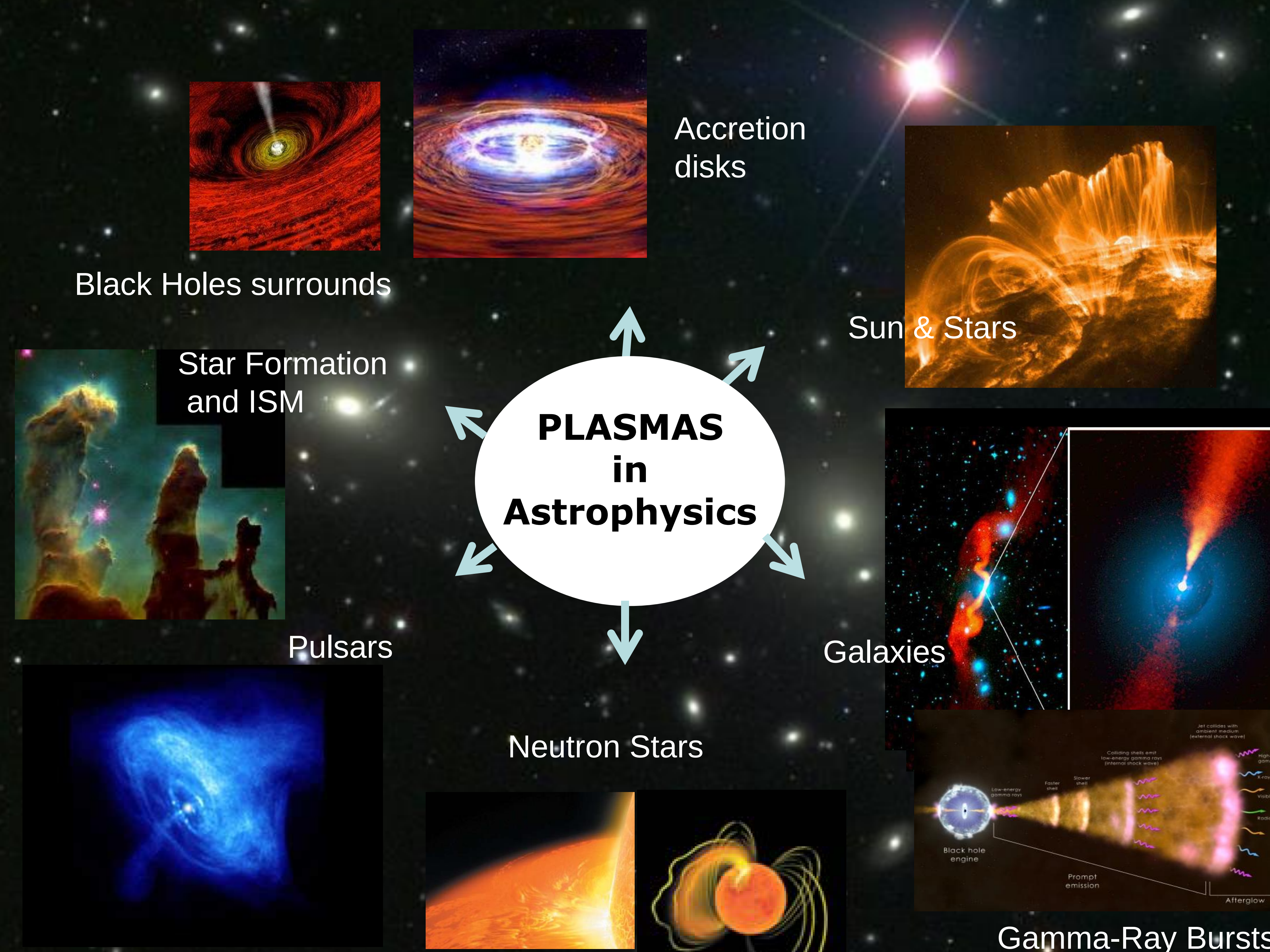
Importance of Plasmas

- **Their importance *not well understood* yet** in:

  - stellar evolution

  - galaxy evolution

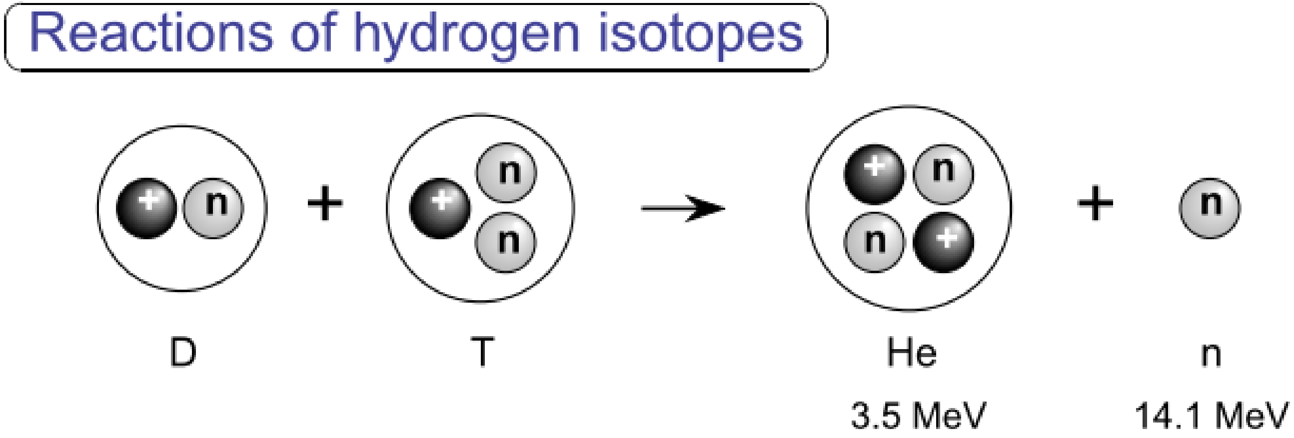
  - structure formation in the early Universe


PLASMAS
in
Astrophysics
Black Holes surrounds
Accretion
disks
Sun & Stars
Star Formation
and ISM
Pulsars
Neutron Stars
Galaxies
Black hole
engine
Prompt
emission
Afterglow
Gamma-Ray Bursts

Plasmas also in Laboratory

Major goal is to produce clean energy via:

- Thermonuclear fusion (**as in the interior of stars**) $\rightarrow$ production of energy!



Magnetic Fields in a PLASMA provide the only way to confine matter of very high temperatures during long times!

Understanding Quasi Neutrality

$$\boxed{Zen_i = en_e}$$

- Let us assume a gas of charged particles with local charge concentration ($e\,\delta n_e$)

→ according to Coulomb law: $$\vec{\nabla} \cdot \vec{E} = \frac{\partial E_{//}}{\partial x} = -4\pi e \delta n_e$$

→ generates the electric field: $$E_{//} = 4\pi e n_e \xi$$

→ $E_{//}$ provokes on the thermal random motion of electrons: flow with velocity $v_{//}$:

$$\frac{dv_{//}}{dt} = \frac{d^2\xi}{dt^2} = -\frac{e}{m_e}E_{//} = -\frac{4\pi n_e e^2}{m_e}\xi$$

- Solution: simple **harmonic motion** with **plasma electron frequency** that in the average neutralizes $E_{//}$:

$$\boxed{\omega_{pe} = \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2} \simeq 5.6 \times 10^4 n_e^{1/2} \quad s^{-1}}$$

Quasi Neutrality- Plasma Frequency

- ω_{pe} **defines natural** **plasma frequency** (neutralizes E effect)
- ions oscilate with much << frequency → (practically at rest in relation to els.)

$$\boxed{\omega_{pi} = \left(\frac{Zm_e}{m_i}\right)^{1/2} \omega_{pe}}$$

- Any external electric field E applied with freq. $\omega < \omega_{pe}$ is **unable to penetrate** the plasma
- **Debye length**: $\boxed{\lambda_D = v_{th}/\omega_{pe}}$

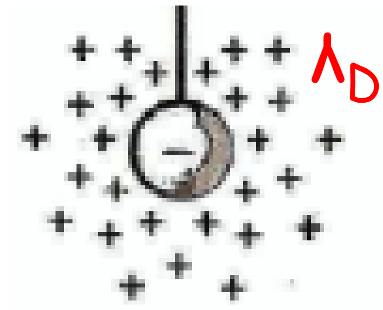
→ E does not penetrate plasma if:

$$\omega < \omega_{pe} \rightarrow \lambda > \lambda_D$$

PLASMA: Debye length (λ_D)

- λ_D → scale within which separation between charges can be "felt":

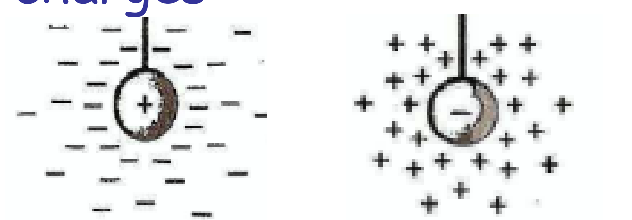
$$\boxed{\lambda_D \simeq \frac{v_{th}}{\omega_{pe}} \simeq \left(\frac{KT_e}{4\pi n_e e^2}\right)^{1/2} = 7\ cm\ (T_e/n_e)^{1/2}}$$



- Within sphere of radius (λ_D) → charge neutrality is not valid: **electrostatic external oscillations with $\lambda \leq \lambda_D$** penetrate the sphere and feel the collective effects of the charges → **strongly damped (Landau damping)**

- Electric Potential Field of a charge within plasma: has its action screened (or partially blocked) by clouds of charges

$$\boxed{\phi(r) = \frac{-en_1}{r}\, e^{-r/\lambda_D}}$$



Plasma Quasi Neutrality

$$\boxed{Zen_i = en_e}$$

IS VALID WHEN:

$$\omega < \omega_{pe} \text{ or } \lambda > \lambda_D$$

Quasi Neutrality in Astrophysical Plasmas

- Tpypical dimension of astrophysical plasmas: $L \gg \lambda_D$

⇨ **quase neutrality is valid**

→ **Internal E fields: little important !**

(neutralized by strong plasma oscillations ω_{pe})

→ **External E fields typically do not penetrate**

($\omega \ll \omega_{pe}$, $\lambda \gg \lambda_D$)

Plasma: collective behaviour

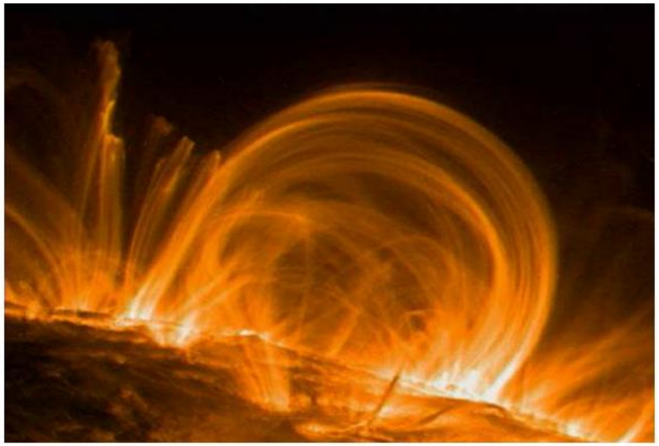
$$\lambda_D \simeq \frac{v_{th}}{\omega_{pe}} \simeq \left(\frac{KT_e}{4\pi n_e e^2}\right)^{1/2} = 7\ cm\ (T_e/n_e)^{1/2}$$

Ex. Solar corona:

$T_e \sim 10^6$ K, $n_e \sim 10^{10}$ cm^{-3}

-> $\lambda \gg \lambda_D \sim 0.07$ cm

Example: Coronal loops (cont'd)



[from recent observations with TRACE spacecraft]

-> So that the system size λ is much larger than λ_D: collective behaviour

Plasma Physics

Three theoretical models:

- **Theory of motion of single charged particles** in given magnetic and electric fields;
- **Kinetic theory of a collection of such particles,** describing plasmas *microscopically* by means of particle distribution functions $f_{e,i}(\mathbf{r}, \mathbf{v}, t)$;
- **Fluid theory (magnetohydrodynamics),** describing plasmas in terms of averaged *macroscopic* functions of $\mathbf{r}$ and t .

Plasma Physics

Three theoretical models:

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CGS

Motion of a single charge

Equation of motion

of charged particle in given electric and magnetic field, $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$:

$$m\frac{d\vec{v}}{dt} = q\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)$$

Let us

- Apply to constant magnetic field $\mathbf{B} = B\mathbf{e}_z$, $\mathbf{E} = 0$:

- Systematic solution of Eq. (1) with $\mathbf{v} = d\mathbf{r}/dt = (\dot{x}, \dot{y}, \dot{z})$ gives two coupled differential equations for motion in the perpendicular plane:

$$\ddot{x} - (qB/mc)\ \dot{y} = 0\,,$$

$$\ddot{y} + (qB/mc)\ \dot{x} = 0\,.$$

$\Rightarrow$ periodic motion about a fixed point $x = x_c, y = y_c$ *(the guiding centre)*.

Motion of a single charge

CGS

Cyclotron motion

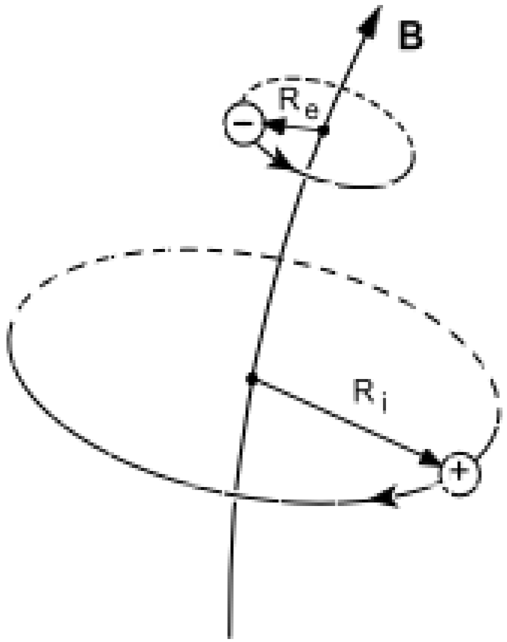
This yields periodic motion in a magnetic field, with *gyro- (cyclotron) frequency*

$$\omega_B = \frac{|\,q\,|\,B}{mc} = \Omega$$

and *cyclotron (gyro-)radius*

$$r_B = \frac{v_\perp}{\omega_B} \approx \frac{\sqrt{2mkT}}{|q|B}c$$

$\Rightarrow$ Effectively, charged particles stick to the field lines.



Opposite motion of electrons and ions about guiding centres with quite different gyro-frequencies and radii, since $m_e \ll m_i$:

$$\Omega_e \equiv \frac{eB}{m_e c} \gg \Omega_i \equiv \frac{ZeB}{m_i c}, \qquad R_e \approx \frac{\sqrt{2m_e kT}}{eB}c \ll R_i \approx \frac{\sqrt{2m_i kT}}{ZeB}c.$$

Motion of a single charge

Cyclotron motion (cont'd)

Orders of magnitude

- Typical gyro-frequencies, e.g. for corona plasma $B \sim 10^4$ G:

$$\Omega_e = 5.3 \times 10^{11}\,\mathrm{rad\,s^{-1}}$$

$$\Omega_i = 2.9 \times 10^{8}\,\mathrm{rad\,s^{-1}}$$

- Gyro-radii, with $v_\perp = v_{\mathrm{th}} \equiv \sqrt{2kT/m}$ for $T_e = T_i \sim 10^6$ K:

$v_{\mathrm{th},e} \sim 3\ 10^8$ cm/s $R_e \sim 10^{-3}$ cm

$v_{\mathrm{th},i} \sim 10^7$ cm/s $\Rightarrow$ $R_i \sim 3\ 10^{-2}$ cm

$\Rightarrow$ The large corona time scales and dimensions *justify averaging.*

Motion of a single charge

- Magnetic fields:
  1. charged particles gyrate around field lines;
  2. fluid and magnetic field move together ("B frozen into the plasma");

Plasma Physics

Three theoretical models:

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Kinetic Description of a Plasma

A plasma consists of a very large number of interacting charged particles $\Rightarrow$ *kinetic plasma theory* derives the equations describing the *collective behavior* of the many charged particles by applying the methods of statistical mechanics.

$$\left(\omega \geq \omega_{pe} \qquad l \geq \lambda_D \right)$$

Kinetic theory describes the collective behaviour of the many charged particles by means of particle distribution functions (**Follker-Planck equation**):

$f(r,u,t)$ ->

$$\boxed{\frac{\partial f}{\partial t} + \mathrm{u} \cdot \nabla_x f + \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathrm{u}} f = 0}$$

Plasma Physics

Three theoretical models:

- **Theory of motion of single charged particles** in given magnetic and electric fields;
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Fluid description: Magnetohydrodynamics (MHD)

Macroscopic definition

Macroscopic model: size and time scales are large enough → possible to apply AVERAGES over microscopic quantities: colective plasma oscillations and collective cyclotron motions of ions and electrons

(a) $\tau \gg \Omega_i^{-1} \sim B^{-1}$ (time scale longer than inverse cyclotron frequency);

(b) $\lambda \gg R_i \sim B^{-1}$ (length scale larger than cyclotron radius).

$\Rightarrow$ MHD $\equiv$ magnetohydrodynamics

PLASMA @ different scales

Intermediate - *microscopic scales* : $l \gg \lambda_D$

kinetic theory describes the collective behaviour of the many charged particles by means of particle distribution functions (**Folker-Planck equation**):

$$f_{e,i}(r,v,t)$$

Large – *macroscopic scales* : $L \gg R_L$

effectively **collisional** -> **fluid description**: size and time scales are large enough → possible to apply AVERAGES over microscopic quantities: over collective plasma oscillations and collective cyclotron motions **- MHD equations**

⬇

Applicable to most astrophysical plasmas

End of Class 1