MHD Formulation

Class 2

MHD Formulation

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Contents

- Fluid description: MHD equations
- Magnetic Force
- Magnetic Flux Freezing

PLASMA @ different scales

Intermediate - *microscopic scales* : $l >> \lambda_D$

kinetic theory describes collective behaviour of many charged particles by means of particle distribution functions:

$$f_{e,i}(r,u,t)$$

Large – *macroscopic scales* : $L >> R_L$

fluid description: size and time scales are large → possible to apply AVERAGES over collective plasma oscillations and cyclotron motions (at each r, t):

MHD description

↓

Applicable to most astrophysical plasmas

Fluid description: MHD

Maxwell eqs. + hydrodynamics eqs. = Eqs. MHD

- **Maxwell's equations** describe evolution of electric field $\mathrm{E}(\mathbf{r}, t)$ and magnetic field $\mathrm{B}(\mathbf{r}, t)$ in response to current density $\mathbf{j}(\mathbf{r}, t)$ and space charge $\tau(\mathbf{r}, t)$:

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} + \nabla\times\mathbf{E} = 0 \qquad \textit{(Faraday)} \qquad (1)$$

$$\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{J} = \nabla\times\mathbf{B} \,, \qquad \textit{('Ampère')} \qquad (2)$$

$$\nabla\cdot\mathbf{E} = 4\pi\rho_{ch} \qquad \textit{(Poisson)} \qquad (3)$$

$$\nabla\cdot\mathbf{B} = 0 \qquad \textit{(no monopoles)} \qquad (4)$$

- **Gas dynamics equations** describe evolution of density $\rho(\mathbf{r}, t)$ and pressure $p(\mathbf{r}, t)$:

$$\frac{\partial n}{\partial t} + \nabla\cdot n\mathbf{v} = 0 \qquad \textit{(mass conservation)} \qquad (5)$$

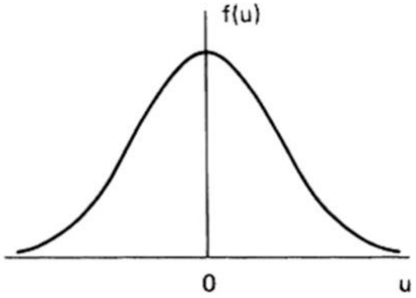
$$nm\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + nq\left(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right) \qquad \textit{(momemtum conservation)} \qquad (6)$$

$$\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla p + \gamma\, p\nabla\cdot\mathbf{v} = 0 \qquad \textit{(energy conservation)} \qquad (7)$$

Key assumption for building MHD eqs.

- Assume the gas achieves local thermal equilibrium

$\longrightarrow$ Maxwellian distribution function:



$$f(u) = A\exp(-\tfrac{1}{2}mu^2/KT)$$

where $f\,du$ is the number of particles per m^3 with velocity between u and $u+du$, $\frac{1}{2}mu^2$ is the kinetic energy, and K is Boltzmann's constant,

$$K = 1.38 \times 10^{-23}\,\mathrm{J/^\circ K} \quad = 1.38\ 10^{-16}\ \mathrm{erg/^oK}$$

The density n, or number of particles per m^3, is given by

$$\boxed{n = \int_{-\infty}^{\infty} f(u)du}$$

And the average velocity of Maxwellian distribution (one-dimension):

$$\boxed{v_{th} = (2KT/m)^{1/2}}$$

Fluid description: MHD

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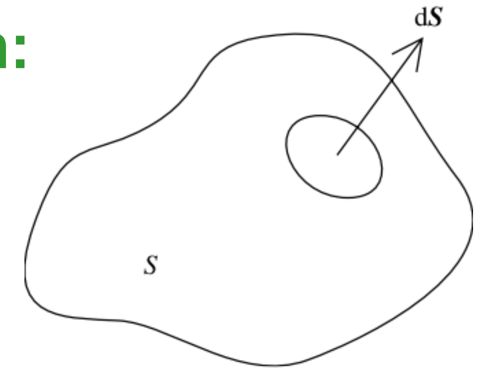
A Glance at Gas Dynamics

Mass Conservation (continuity) equation:



- ✓ Fixed volume V and surface S around it
- ✓ Rate of change of mass in the fluid contained in V is:

$\rho = n\, m$ $$\frac{\partial}{\partial t}\int_V \rho\, \mathrm{d}V.$$

In the absence of sources or sinks of matter, this is = to the **net inflow of mass** over the whole surface.

- ✓ **Outward mass flow** across an element area in that time is $\rho\, \mathbf{u} \cdot \mathrm{d}\mathbf{S}$.
- ✓ Hence mass gained by volume V is $-\int_S \rho\, \mathbf{u} \cdot \mathrm{d}\mathbf{S} = -\int_V \boldsymbol{\nabla} \cdot (\rho\, \mathbf{u})\, \mathrm{d}V,$ (Gauss theorem)
- ✓ Therefore $\frac{\partial}{\partial t}\int_V \rho\, \mathrm{d}V = -\int_V \boldsymbol{\nabla} \cdot (\rho\, \mathbf{u})\, \mathrm{d}V.$
- ✓ Since this is true for all volumes ->

$$\boxed{\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho\, \mathbf{u}) = 0. \qquad \mathit{Eulerian}}$$

A Glance at Gas Dynamics

Fluid equations for each charged particle species:

$\rho = nm$ $$\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{v} = 0$$

Mass conservation

$$nm\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + nq\left(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right)$$

Momentum conservation

$$\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla p + \gamma\ p\nabla\cdot\mathbf{v} = 0$$

Energy conservation

Where energy equation comes from thermodynamics (*$d\varepsilon + pdV = 0$*), **for IDEAL GAS**:

$$\boxed{\frac{3}{2}nk_B\frac{dT}{dt} + p\nabla\cdot\mathbf{v} = 0}$$

$k_B = K$

Equation of state: $$\boxed{p = nk_BT}$$

γ: ratio of specific heats

A Glance at Gas Dynamics

Eulerian and Lagrangean Derivatives:

- ✓ Global fixed frame *(t,x,y,z)*, and co-moving frame *(t',x',y',z')*

- ✓ Local transformation:

$$dx' = dx - v_x dt \qquad dy' = dy - v_y dt \qquad dz' = dz - v_z dt$$

The time derivative transforms as :

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{dx}{dt}\frac{\partial}{\partial x} + \frac{dy}{dt}\frac{\partial}{\partial y} + \frac{dz}{dt}\frac{\partial}{\partial z}$$
$$= \frac{\partial}{\partial t} + v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z}$$

$$\Longrightarrow$$

Lagrangean time derivative (moving with fluid)

$$\boxed{\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla}$$

- ✓ Therefore, two forms of **mass conservation equation**:

$$\boxed{\frac{dn}{dt} + n\nabla\cdot\mathbf{v} = \frac{\partial n}{\partial t} + \nabla\cdot n\mathbf{v} = 0}$$

$$\boxed{n\,m = \rho}$$

Glance at Lagrangean & Eulerian Derivatives

$$\frac{D}{Dt} = \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

- dh/dt: ***DOES NOT*** refer to the rate of change of h at a fixed point of space (local derivative),
- dh/dt: is the rate of change of h in a portion of the fluid when moving in space (material derivative – co-moving with flow).
- dh/dt has two contributions:
  - one that originates from variation of h over time at a fixed point (r) of the space ($\partial h/\partial t$)
  - The other that originates from the difference of h between two points of the fluid separated by a distance dr at fixed time ($v.\nabla h$)

    (which actually corresponds to the distance that the portion of the fluid moves during time dt)

A Glance at Gas Dynamics

Lagrangean formulation

$$\frac{D}{Dt} = \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

- **Gas dynamics equations** describe evolution of density $\rho(\mathbf{r}, t)$ and pressure $p(\mathbf{r}, t)$:

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} + \rho \nabla \cdot \mathbf{v} \equiv \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0\,,$$ *(mass conservation)*

$$\frac{\mathrm{D}p}{\mathrm{D}t} + \gamma p \nabla \cdot \mathbf{v} \equiv \frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \gamma p \nabla \cdot \mathbf{v} = 0\,,$$ *(energy conservation)*

and the *momentum equation*:

$$nm \frac{d\vec{v}}{dt} = nm \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + nq \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right)$$

Building the MHD eqs.

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Building the MHD eqs.

One-fluid approximation

Combining eqs. of motion of eletrons and ions:

- Define one-fluid variables that are linear combinations of the two-fluid variables:

$$\rho \equiv n_e m_e + n_i m_i \,, \qquad \text{(total mass density)}$$

$$\rho_{ch} \equiv -e\,(n_e - Z n_i) = 0 \qquad \text{(charge density)}$$

$$\mathbf{v} \equiv (n_e m_e \mathbf{u}_e + n_i m_i \mathbf{u}_i)/\rho \,, \qquad \text{(center of mass velocity)}$$

$$\mathbf{j} \equiv -e\,(n_e \mathbf{u}_e - Z n_i \mathbf{u}_i) \,, \qquad \text{(current density)}$$

$$p \equiv p_e + p_i \,. \qquad \text{(pressure)}$$

✓ **Neutral current carrying gas:**
ions carry mass, momentum, energy
electrons carry current, thermal energy

Assume "slow" dynamics:
Finite light speed c ignored
1/t = w << c/L
Neglect terms : $c^{-1}(\partial/\partial t)$

Building the MHD Equations

CGS

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0\,, \qquad \textit{(continuity)}$$

$$\rho\,(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}) + \nabla p - \boxed{\frac{1}{c}\vec{J}\times\vec{B}} = 0\,, \qquad \textit{(momentum)}$$

$$\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla p + \gamma p \nabla\cdot\mathbf{v} = (\gamma-1)\eta|\mathbf{j}|^2\,, \qquad \textit{(internal energy)}$$

$$\frac{\partial \mathbf{B}}{\partial t} + c\vec{\nabla}\times\vec{E} = 0\,, \qquad \textit{(Faraday)}$$

where

$$\vec{J} = \frac{c}{4\pi}\vec{\nabla}\times\vec{B} \qquad \textit{(Ampère)}$$

$$\vec{E} = -\frac{\vec{v}_e}{c}\times\vec{B} + \frac{m_e}{e}\vec{g} - \frac{1}{n_e e}\vec{\nabla}p_e + \boxed{\eta\vec{J}} \qquad \textit{(Ohm)}$$

and

$$\nabla\cdot\mathbf{B} = 0 \qquad \textit{(no magnetic monopoles)}$$

is initial condition on Faraday's law.

Electric Resistivity: $\eta = \dfrac{m_e v_{th}\sigma_{ei}}{Ze^2}$

For hydrogen gas (Z=1)

$$\eta = \frac{7\times 10^{-9}}{T^{3/2}}\ln\Lambda \quad s$$

lnΛ=20-30 (astrophysical plasmas)

Magnetic Induction Equation

$$\frac{\partial \vec{B}}{\partial t} = -c\vec{\nabla} \times \vec{E}$$

$$\vec{E} = -\frac{\vec{v}_e}{c} \times \vec{B} + \frac{m_e}{e}\vec{g} - \frac{1}{n_e e}\vec{\nabla} p_e + \eta \vec{J}$$

$$\boxed{\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v}_e \times \vec{B}) + \frac{\eta c^2}{4\pi}\vec{\nabla}^2 \vec{B} - \frac{c}{n_e^2 e}\vec{\nabla} n_e \times \vec{\nabla} p_e}$$

Advection: gas and magnetic field coupling (freezing)

Viscosity: dissipation of the magnetic field

Biermann Battery: only important for generation of B (dynamos) -> non null differential rotation (in general neglected)

MHD Equations: ± usual

CGS

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0\,,$$ *(continuity)*

$$\rho\left(\frac{\partial\mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) + \nabla p - \boxed{\frac{1}{c}\vec{J}\times\vec{B}} = 0\,,$$ *(momentum)*

$$\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla p + \gamma p\nabla\cdot\mathbf{v} = (\gamma-1)\eta|\mathbf{j}|^2\,,$$ *(internal energy)*

$$\frac{\partial\vec{B}}{\partial t} = \vec{\nabla}\times(\vec{v}_e\times\vec{B}) + \boxed{\nu_M}\vec{\nabla}^2\vec{B}$$ ***(magnetic induction)***

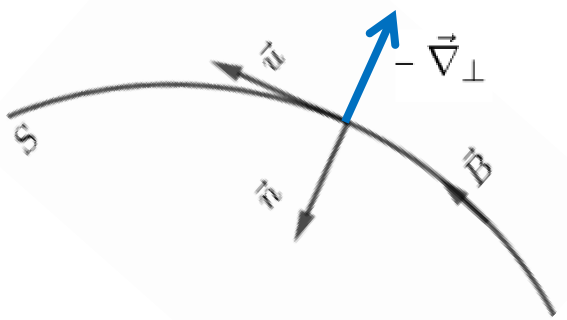
$$\vec{J} = \frac{c}{4\pi}\vec{\nabla}\times\vec{B} \qquad \nabla\cdot\mathbf{B} = 0\,.$$ *(Ampere, divergencia)*

Where **magnetic resistivity**: $\boxed{\nu_M = \frac{\eta c^2}{4\pi}}$ **(cm²/s)**

Eq. of state to close the system: $\boxed{p = nk_BT}$ (if ideal gas)

Magnetic Force

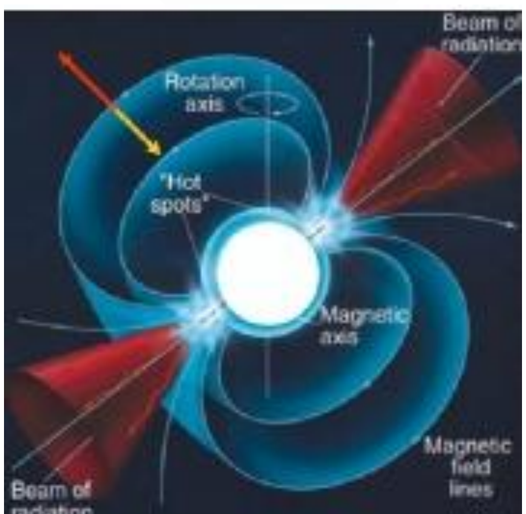
$$\frac{1}{c}\vec{J}\times\vec{B} = \frac{1}{4\pi}(\vec{\nabla}\times\vec{B})\times\vec{B} = \frac{1}{4\pi}(\vec{B}.\vec{\nabla})\vec{B} - \vec{\nabla}\left(\frac{B^2}{8\pi}\right)$$



$$\boxed{\frac{1}{c}(\vec{J}\times\vec{B}) = \frac{B^2}{4\pi}\frac{\vec{n}}{\lambda} - \vec{\nabla}_\perp\left(\frac{B^2}{8\pi}\right)}$$

Tension Force:
force directed towards center of curvature -> *field lines as "wires with tension"*

Magnetic Pressure Force



Ex.: Dipole magnetic field of a star magnetosphere (as in a pulsar):

tension = magnetic pressure

$$\frac{1}{c}(\vec{J}\times\vec{B}) = 0$$

IDEAL MHD Concept

$$\boxed{\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v}_e \times \vec{B}) + \nu_M \vec{\nabla}^2 \vec{B}}$$

Ratio between these two terms:

$$\boxed{\frac{|\vec{\nabla} \times (\vec{v}_e \times \vec{B})|}{\nu_M |\vec{\nabla}^2 \vec{B}|} \sim \frac{L v_e}{\nu_M} = R_{eM}}$$

→ **Magnetic Reynolds number**

In **astrophysical plasmas** in general: $R_{eM} >> 1$ → **ideal MHD:**

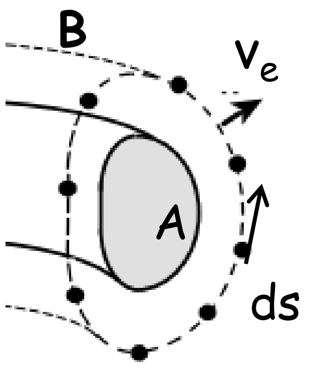
→ $$\boxed{\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v}_e \times \vec{B}) + \cancel{\nu_M \vec{\nabla}^2} \vec{B}}$$

Exceptions: $R_{eM} \approx 1$: Ex. **Magnetic Reconnection** → **resistive MHD**

Ideal MHD → B Flux Freezing

With η = 0: $\boxed{\nu_M = \frac{\eta c^2}{4\pi} = 0}$ → $\boxed{\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v}_e \times \vec{B})}$

Integrating over an open surface A surrounded by a closed contour ∂S and using Stokes' theorem:



$$\frac{\partial}{\partial t}\int_A \vec{B}.d\vec{A} + \oint_{\partial S} (\vec{v}_e \times d\vec{s}).\vec{B} = 0$$

→ The magnetic flux through A with closed contour that moves with the electron gas is CONSTANT (if perfectly conducting fluid)

→ **Concept of flux freezing -> eq. above equivalent to:**

$$\boxed{\frac{d}{dt}\phi = 0} \qquad \boxed{\phi = \int \vec{B}.d\vec{A}}$$

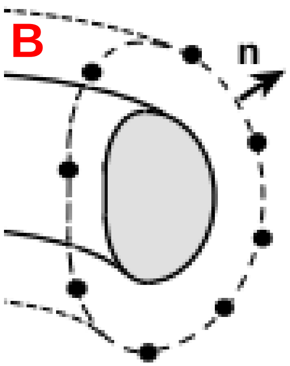
Where d/dt is comoving (Lagrangean) derivative: $\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}_e.\vec{\nabla}$

B Flux Freezing: ideal MHD

$$\boxed{\frac{d}{dt}\phi = 0}$$

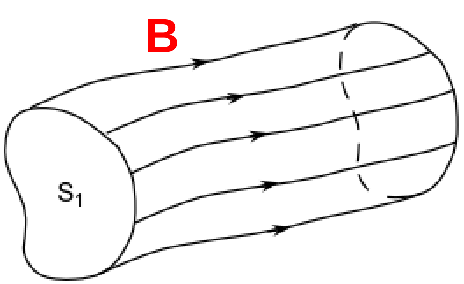
→ **Magnetic flux freezing (flux of B in co-moving area with the flow is constant):**

• It means we can see the lines of force of B as "frozen" in the electron gas and moving along with the gas



• Any motion transverse to the lines of force of the magnetic field, carries them along with the fluid

• A fluid element that moves along a **flux tube** remains moving with it.



In ideal MHD, the magnetic field and plasma are *frozen-in* to each other

- If a parcel of plasma moves, the magnetic field attached to the parcel moves along with it
- More rigorously: if two plasma elements are initially connected by a magnetic field line, they will remain connected by a magnetic field line at future times.
- Magnetic topology (e.g., connectivity) is preserved in ideal MHD
- The plasma cannot move across magnetic field lines (though it remains free to move along the field)

IDEAL MHD Equations

CGS

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0\,, \qquad \textit{(continuity)}$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) + \nabla p - \boxed{\frac{1}{c}\vec{J}\times\vec{B}} = 0\,, \qquad \textit{(momentum)}$$

$$\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla p + \gamma p \nabla\cdot\mathbf{v} = \cancel{(\gamma-1)\eta|\mathbf{j}|^2}\,, \qquad \textit{(internal energy)}$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla}\times(\vec{v}_e\times\vec{B}) + \cancel{\boxed{\nu_M}\vec{\nabla}^2\vec{B}} \qquad \textit{(magnetic induction)}$$

$$\vec{J} = \frac{c}{4\pi}\vec{\nabla}\times\vec{B} \qquad \nabla\cdot\mathbf{B} = 0\,. \qquad \textit{(Ampere, divergencia)}$$

Where **magnetic resistivity**: $\boxed{\nu_M = \frac{\eta c^2}{4\pi}}$ **(cm^2/s)**

Eq. of state to close the system: $\boxed{p = n k_B T}$ (if ideal gas)

IDEAL MHD Equations

CGS

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0\,,$$ *(continuity)*

$$\rho\,(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}) + \nabla p - \boxed{\frac{1}{c}\vec{J} \times \vec{B}} = 0\,,$$ *(momentum)*

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \gamma p \nabla \cdot \mathbf{v} = 0\,,$$ *(internal energy)*

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v}_e \times \vec{B})\,.$$ ***(magnetic induction)***

$$\vec{J} = \frac{c}{4\pi}\vec{\nabla} \times \vec{B} \qquad \nabla \cdot \mathbf{B} = 0\,.$$ *(Ampere, divergencia)*

Eq. of state to close the system:

$$\boxed{p = n k_B T}$$ (if ideal gas)

Is B flux freezing always valid?

→ In astrophysical plasmas: **flux freezing valid in general because**

$L, v \gg 1 \rightarrow \frac{L v_e}{\nu_M} = R_{eM} \gg 1$

- **BUT there are exceptions:**

Ex. 1) **magnetic reconnection**: field dissipation (solar corona, earth magnetosphere)

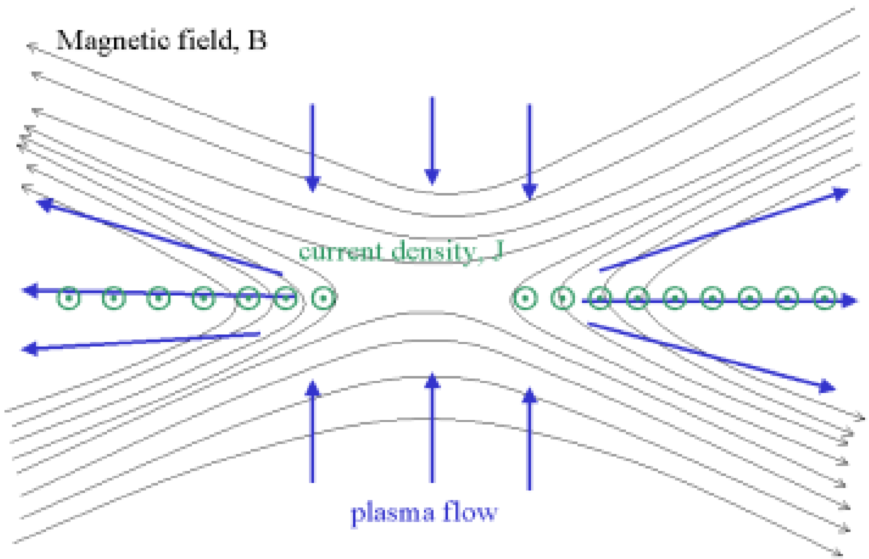
Ex. 2) **MHD turbulence**: wandering of lines -> reconnection

Ex. 3) **dynamos**: magnetic field generation

Is B flux freezing always valid?

→ NO

Ex. 1) **magnetic reconnection sites**: B flux does not conserve because



Example: Coronal loops (cont'd)

[from recent observations with TRACE spacecraft]

$$\frac{L v_e}{\nu_M} = R_{eM} \quad \sim 1$$

$$\boxed{\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v}_e \times \vec{B}) + \nu_M \vec{\nabla}^2 \vec{B}}$$

Is B flux freezing always valid?

→ NO

Ex. 2) collapse of an interstellar cloud to form a star:

Cloud:
$\rho \sim 10^{-20}\,\mathrm{g cm^{-3}}$
$B \sim 10^{-4}\,G$
$R \sim 10^7\,R_*$

Star:
$\rho_* \sim 1\,\mathrm{g cm^{-3}}$
$B_* = ??$

If we use ideal MHD → **B flux conservation** : $\frac{d}{dt}\phi = 0 \qquad \phi = \int \vec{B}.d\vec{A}$
+ **mass conservation eq.**, we obtain:

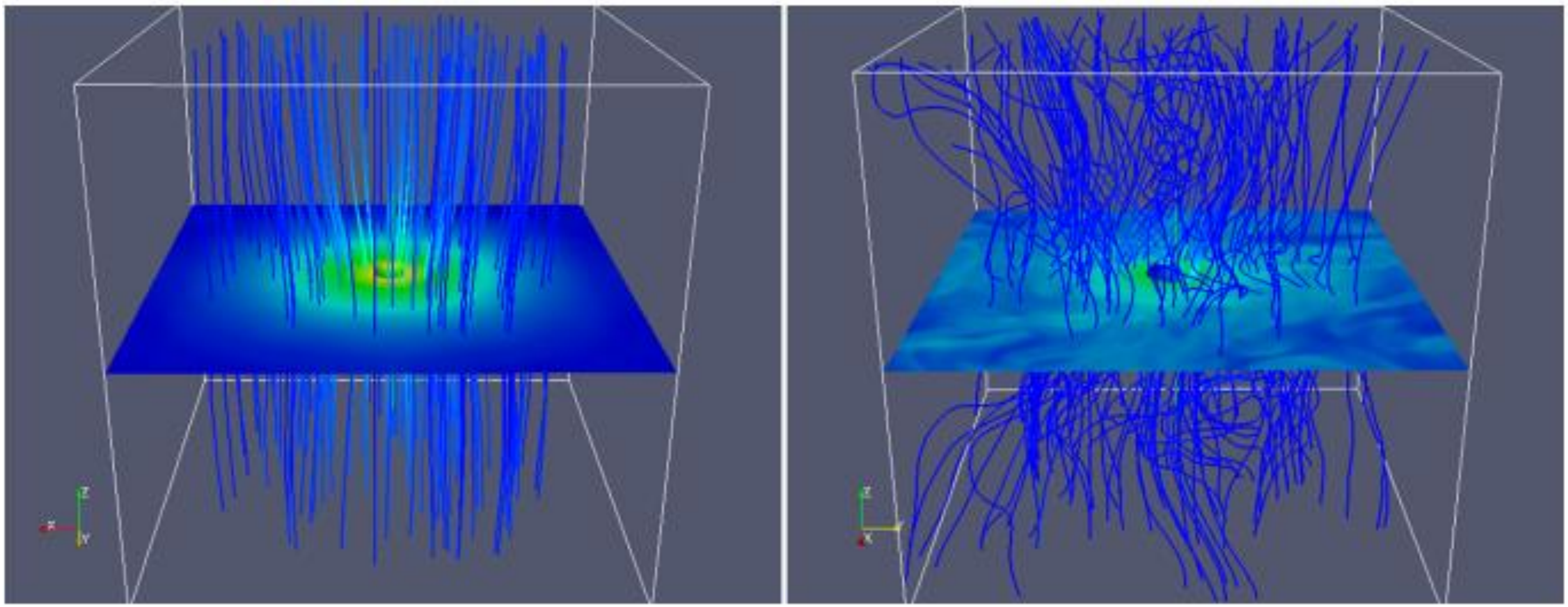
$B_* \sim 10^9\,G$!

BUT, observations: $B_* \sim 10^3\,G$

Therefore: There was no flux conservation! There were flux removal. **What resistive process did that?**

Self-Gravitating collapsing clouds

Non-turbulent t~ 40Myr Turbulent

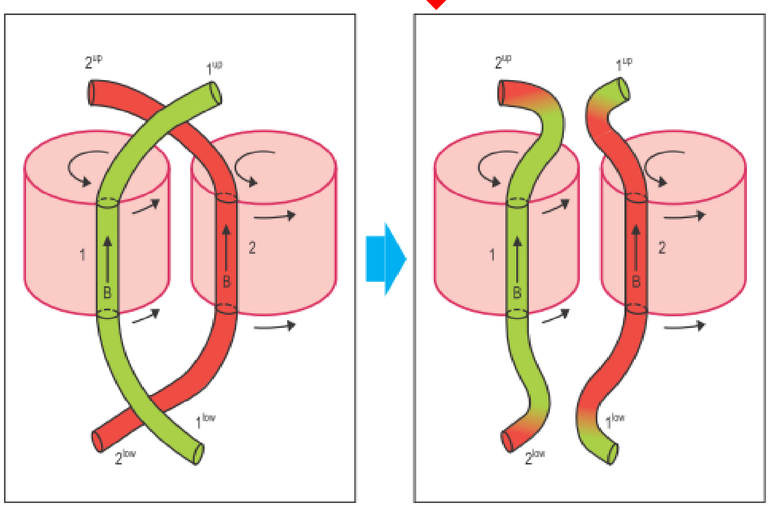


B does not allow core collapse **But with turbulence it collapses**

Leão, de Gouveia Dal Pino et al., ApJ 2013

MHD turbulent diffusion: new scenario

In presence of turbulence: field lines reconnect fast (Lazarian & Vishniac 1999) and magnetic flux transport becomes efficient



Diffusion coefficient:

$$\boxed{\eta_t \sim l_{\rm inj} v_{\rm turb}}$$

Lazarian 2005, 2012
Santos-Lima et al. 2010, 2012, 2013
de Gouveia Dal Pino et al. 2012

Is B flux freezing always valid?

→ NO

Ex. 3) **dynamo**: generates magnetic fields: obviously does not conserve magnetic flux → NON IDEAL MHD

$$\boxed{\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v}_e \times \vec{B}) + \frac{\eta c^2}{4\pi}\vec{\nabla}^2\vec{B} - \frac{c}{n_e^2 e}\vec{\nabla} n_e \times \vec{\nabla} p_e}$$

....+ new terms

Biermann Battery

References

- E. M. de Gouveia Dal Pino "Course Notes" and bibliography therein
- H. Goedbloed notes "*Principles of MHD*"
- *C. Clarke & N. Carswell "Principles of Astrophysical Fluids"*

End of Class 2