41th International School of Young Astronomers
ISYA 2018

Stellar Structure, Evolution and Atmospheres

Karla Peña Ramírez
Universidad de Antofagasta, Chile

Stellar Structure

- Stellar structure equations
- Energy transport: conduction and radiation
- Opacities
- Equation of state
- Nuclear energy generation and burning stages
- Energy transport: convection
- Stellar interiors

Stellar Evolution

- Schematic evolution of stars
- Pre main sequence and main sequence
- Post main sequence

Stellar Atmospheres

- Spectral classification: definitions
- Equations of conservation and radiative transport
- Atomic processes and opacities
- Static equilibrium equations
- Spectral line profiles
- Atmosphere models

+ group hands-on activities

+ paper review

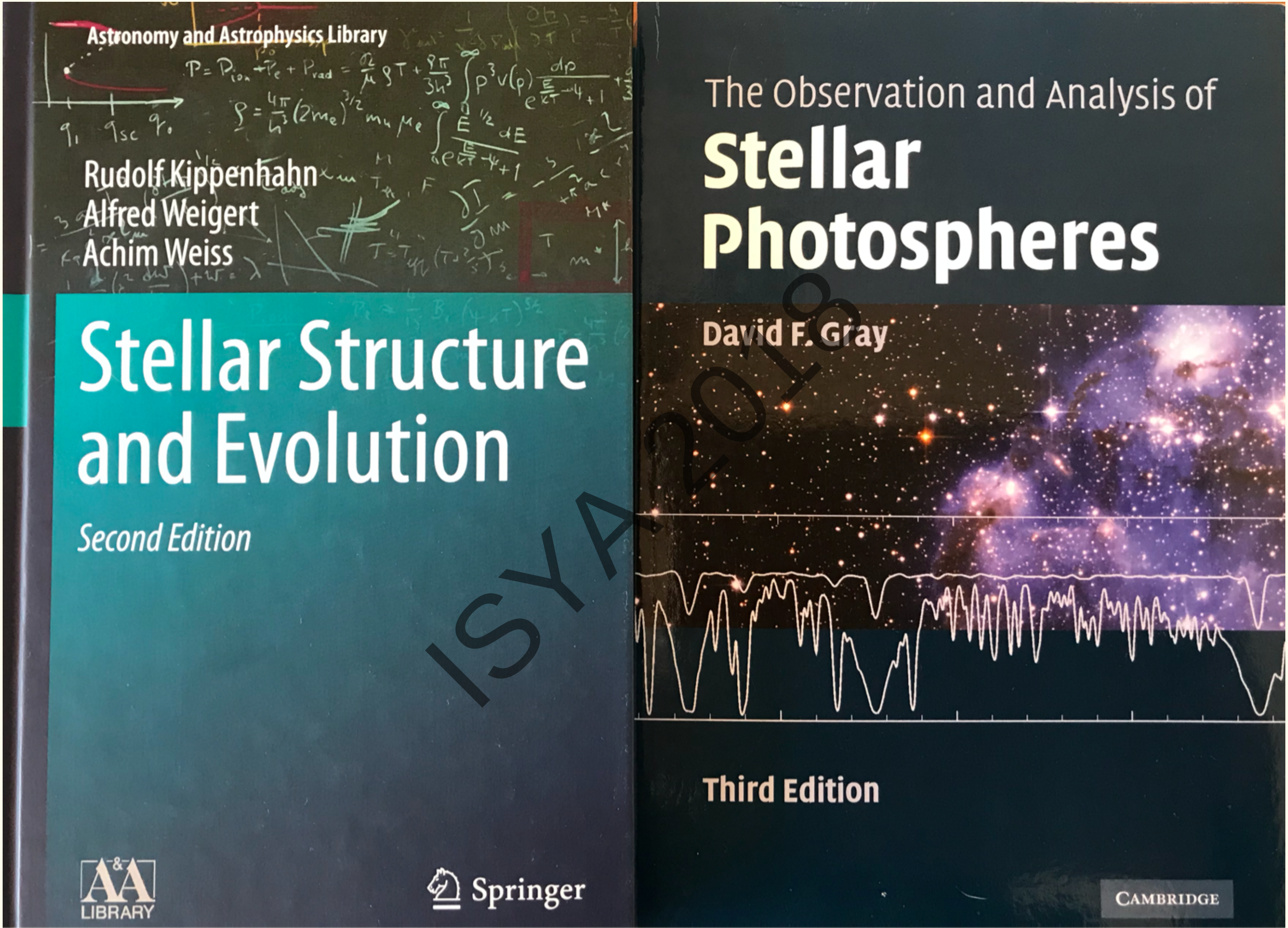
Goals:

1. Broad review of structure and physics of stars.

2. Familiarization with tools/resources used in stellar astrophysics research.

3. Familiarization with state-of-art research in this field.

4. Developing scientific skills: literature research, quantitative analysis, estimation, scientific writing, scientific presentation.


Astronomy and Astrophysics Library
Rudolf Kippenhahn
Alfred Weigert
Achim Weiss
Stellar Structure and Evolution
Second Edition
A&A LIBRARY
Springer
The Observation and Analysis of
Stellar Photospheres
David F. Gray
Third Edition
CAMBRIDGE

Astronomy & Astrophysics manuscript no. mdwarf˙v13
March 16, 2015

New evolutionary models for pre-main sequence and main sequence low-mass stars down to the hydrogen-burning limit

Isabelle Baraffe[1,2], Derek Homeier[2], France Allard[2], and Gilles Chabrier[2,1]

[1] University of Exeter, Physics and Astronomy, EX4 4QL Exeter, UK (e-mail: i.baraffe@ex.ac.uk)
[2] École Normale Supérieure, Lyon, CRAL (UMR CNRS 5574), Université de Lyon, France (e-mail: derek.homeier@ens-lyon.fr, fallard@ens-lyon.fr, chabrier@ens-lyon.fr)

ABSTRACT

We present new models for low-mass stars down to the hydrogen-burning limit that consistently couple atmosphere and interior structures, thereby superseding the widely used BCAH98 models. The new models include updated molecular linelists and solar abundances, as well as atmospheric convection parameters calibrated on 2D/ namics simulations. Comparison of these models with observations in various colour-magnitud ages shows significant improvement over previous generations of models. The new models ca present in the previous ones, such as the prediction of optical colours that are too blue compa vations. They can also reproduce the four components of the young quadruple system LkCa 3 diagram with one single isochrone, in contrast to any presently existing model. In this paper we for consistency when comparing models and observations, with the necessity of using evolution based on the same atmospheric structures.

Key words. stars: low-mass - stars: evolution - stars: pre-main sequence - stars: Hertzsprung-Rus diagrams - convection

AN EMPIRICAL TEMPLATE LIBRARY OF STELLAR SPECTRA FOR A WIDE RANGE OF SPECTRAL CLASSES, LUMINOSITY CLASSES, AND METALLICITIES USING SDSS BOSS SPECTRA

Aurora Y. Kesseli[1], Andrew A. West[1], Mark Veyette[1], Brandon Harrison[1], Dan Feldman[1], John J. Bochanski[2]

(Accepted February 13, 2017)
Draft version March 23, 2017

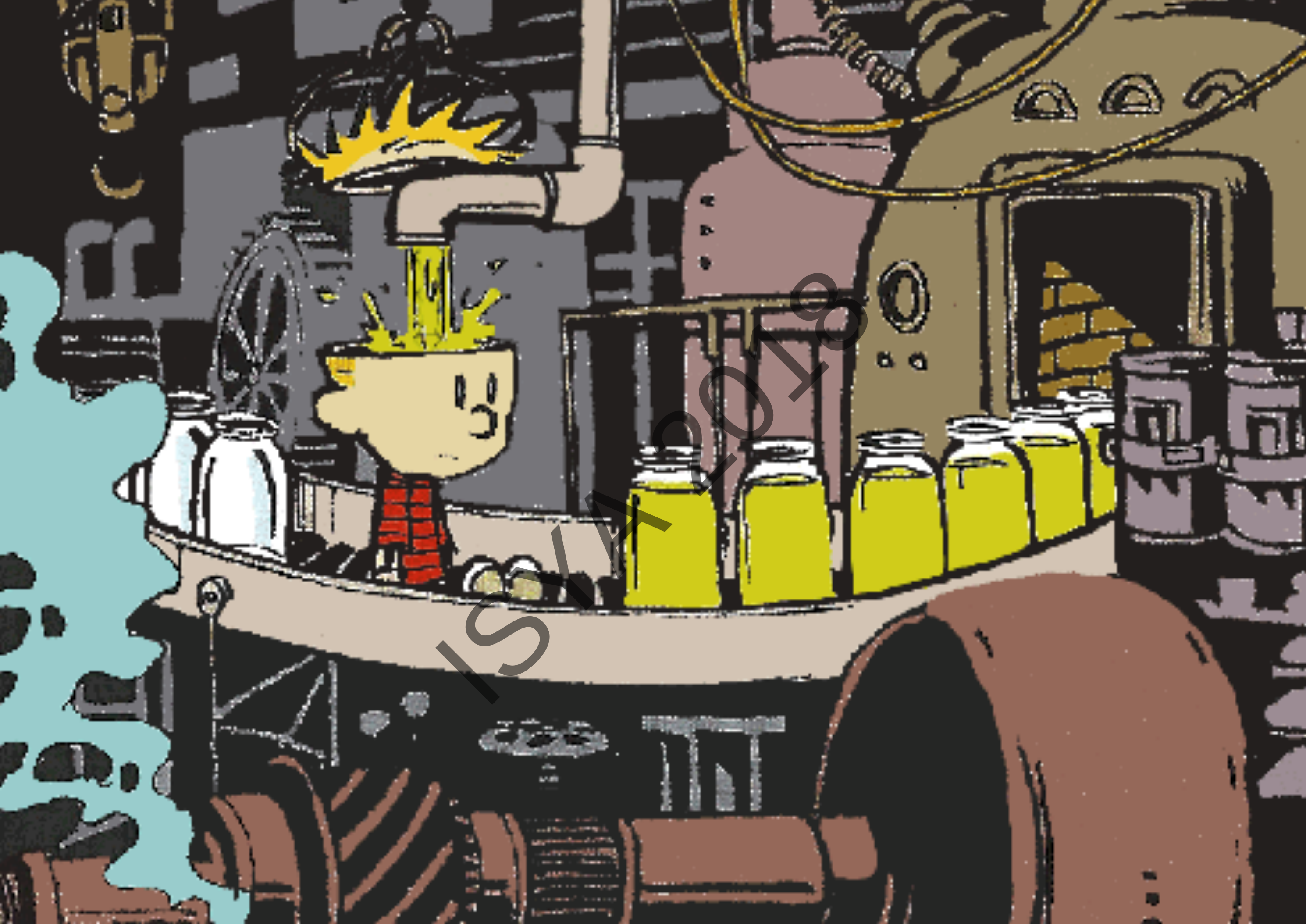
ABSTRACT

We present a library of empirical stellar spectra created using spectra from the Sloan Digital Sky Survey's Baryon Oscillation Spectroscopic Survey (BOSS). The templates cover spectral types O5 through L3, are binned by metallicity from -2.0 dex through +1.0 dex and are separated into main sequence (dwarf) stars and giant stars. With recently developed M dwarf metallicity indicators, we are able to extend the metallicity bins down through the spectral subtype M8, making this the first empirical library with this degree of temperature *and* metallicity coverage. The wavelength coverage for the templates is from 3650 Å through 10200 Å at a resolution better than R$\sim$ 2000. Using the templates, we identify trends in color space with metallicity and surface gravity, which will be useful for analyzing large data sets from upcoming missions like LSST. Along with the templates, we are releasing a code for automatically (and/or visually) identifying the spectral type and metallicity of a star.

Astronomy & Astrophysics manuscript no. DR2HRD
April 26, 2018

Gaia Data Release 2: Observational Hertzsprung-Russell diagrams

Gaia Collaboration, C. Babusiaux[1,2], F. van Leeuwen[3], M.A. Barstow[4], C. Jordi[5], A. Vallenari[6], D. Bossini[6], A. Bressan[7], T. Cantat-Gaudin[6,5], M. van Leeuwen[3], A.G.A. Brown[8], T. Prusti[9], J.H.J. de Bruijne[9], C.A.L.



Star: A. Bound by self-gravity

B. Radiates energy supplied by an internal source.

$\overset{A+B}{\text{They must evolve!}} \xrightarrow{t}$ Changes in structure and composition

Effects on stellar populations

Clusters/Association

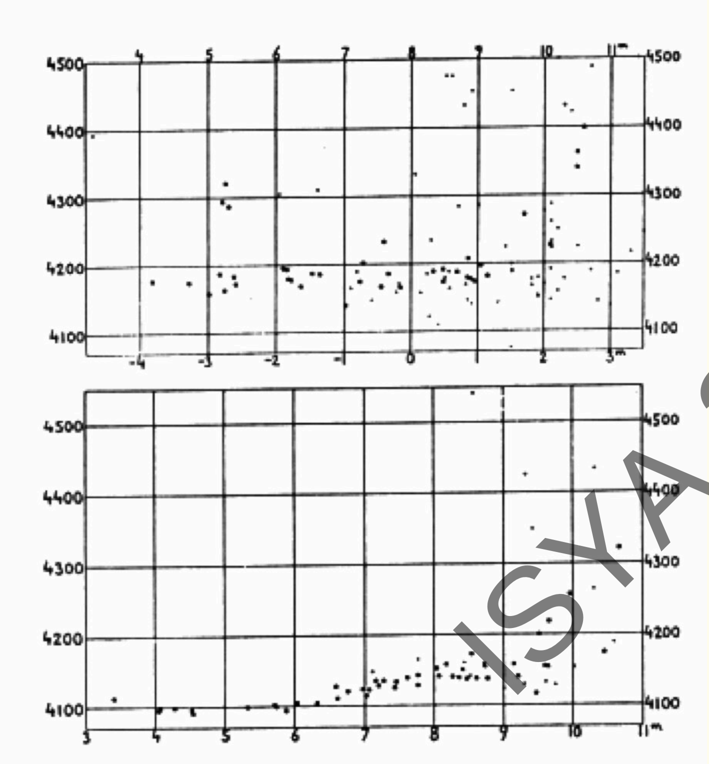
Galaxies

$\xrightarrow{t_{death}}$ A. Material scattering

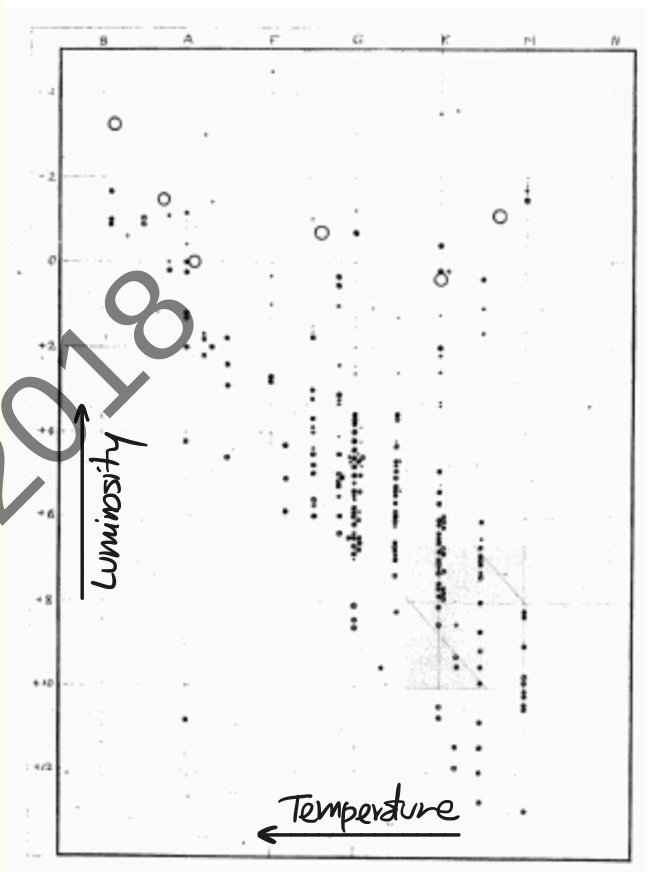
B. Exhaustion of nuclear fuel

$\xleftarrow{t_{birth}}$ Still open problem.

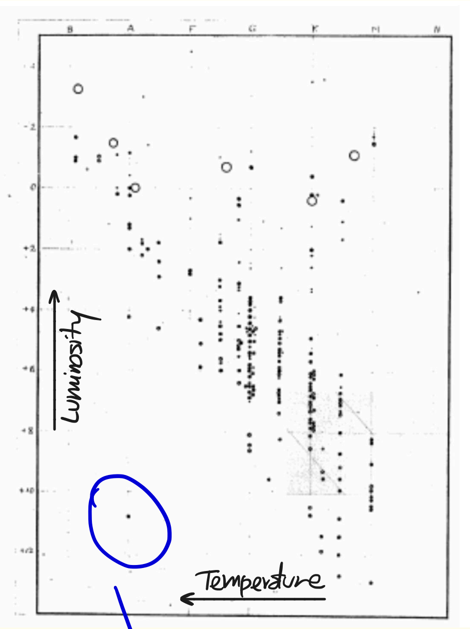
Observational Aspects:



Hertzsprung 1911
Publikationen des Astrophysikalischen Observatorium zu Postdam



Russell 1914
Popular Astronomy, 22, 275



Triple system
K-type star
+
M4.5 flare
+
White Dwarf

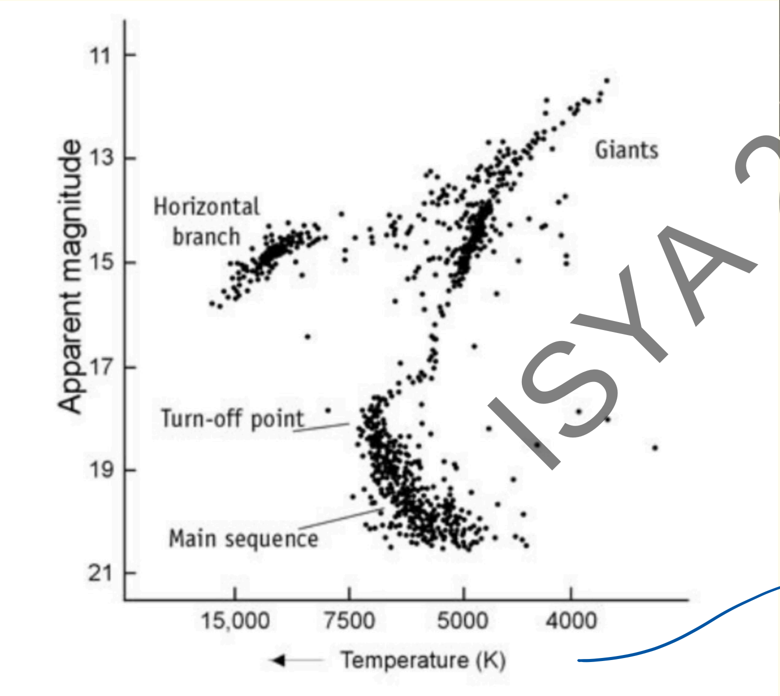
40 Eridani

Multiplicity

M12 in Ophiuchus

$d = 4800$ pc

$t = 12.7$ Gy

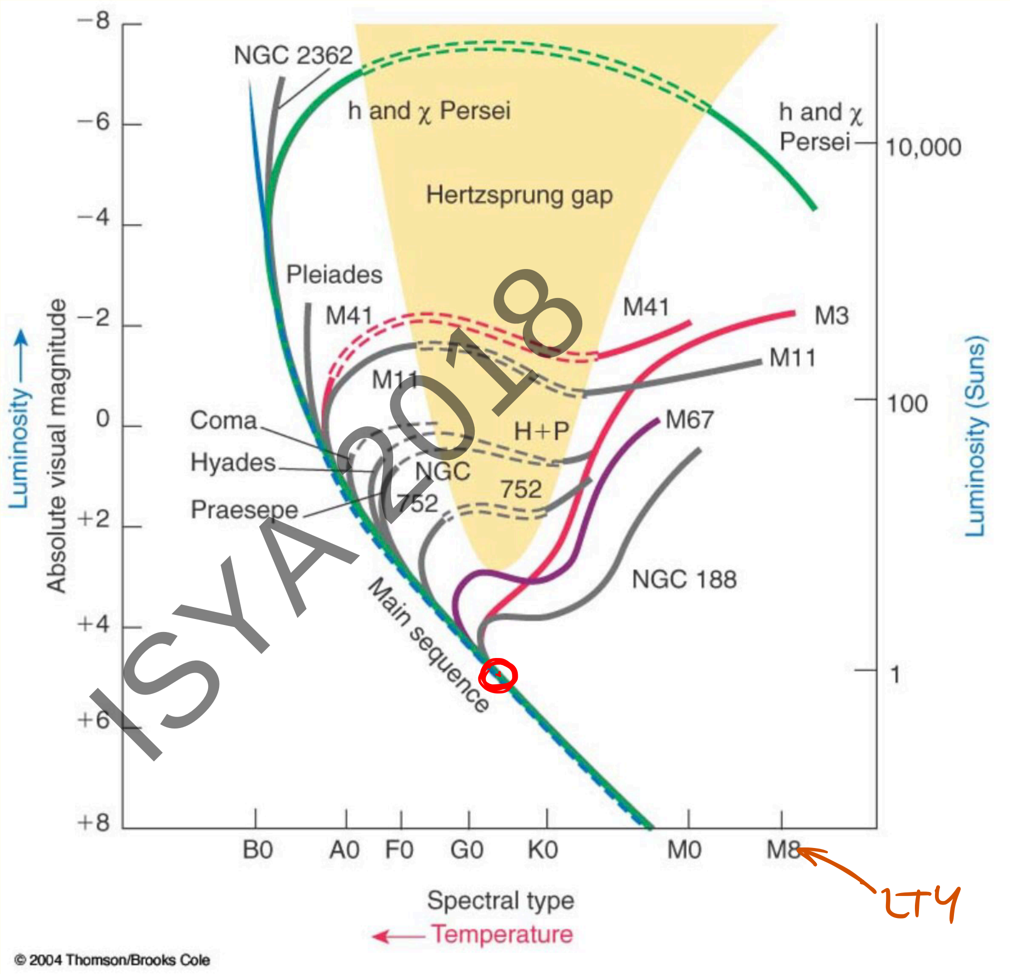


Color index (B-V)

Clusters are coeval !

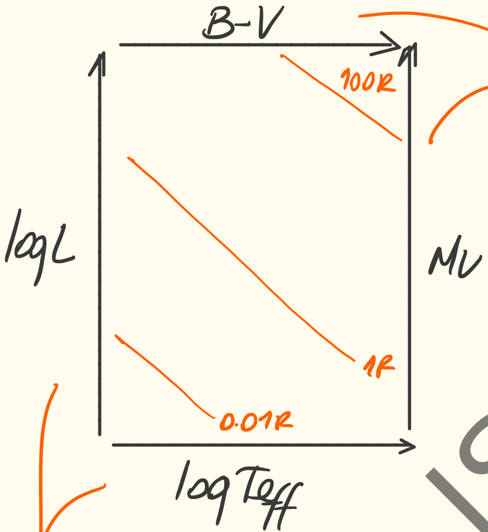
NGC 2362
h and χ Persei
Pleiades
M41
M11
Coma
Hyades
Praesepe
NGC 752
M67
M3
NGC 188

Age



Stellar Structure

Internal structure + Relation with observables (mass, size, luminosities...)

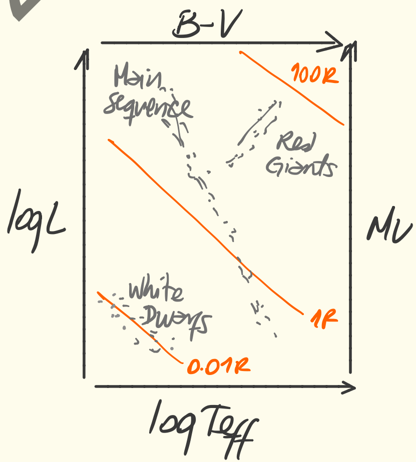


Observables

$$L = 4\pi R^2 \sigma T_{eff}^4$$

$$\log L = 4 \log T_{eff} + 2 \log R + ct$$

Inferred Parameters



Fundamental Parameters: mass + composition

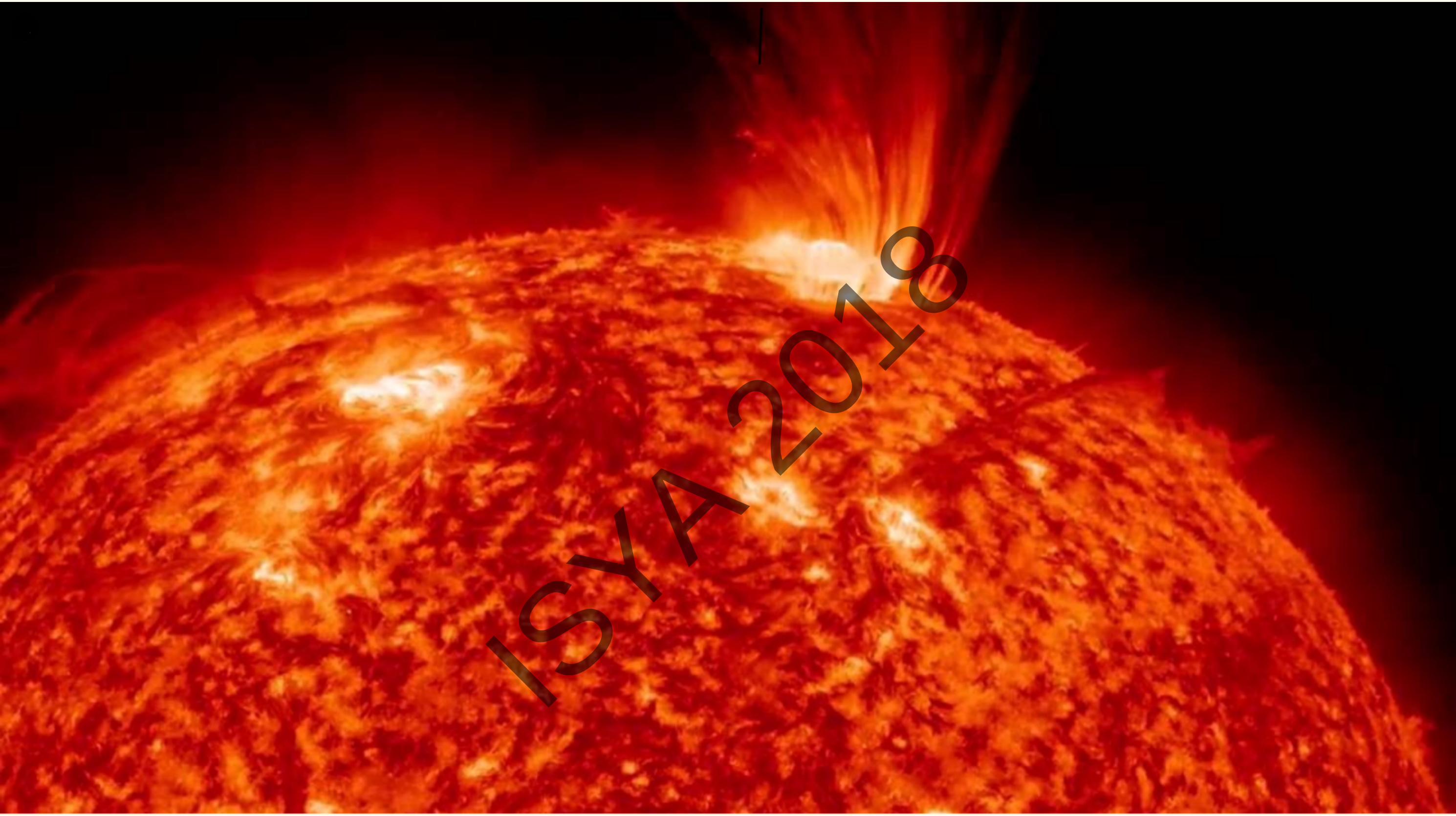
Interior parameters: density, pressure, temperature, luminosity $\longrightarrow$ vary with r.

Component parameters: opacity, heat capacity, energy generation $\longrightarrow$ properties of material.

Boundary conditions: radius, luminosity, effective temperature $\rightarrow$ observables

Secondary parameters: rotation, $\underline{B}$, external forces $\longrightarrow$ extreme cases affected.

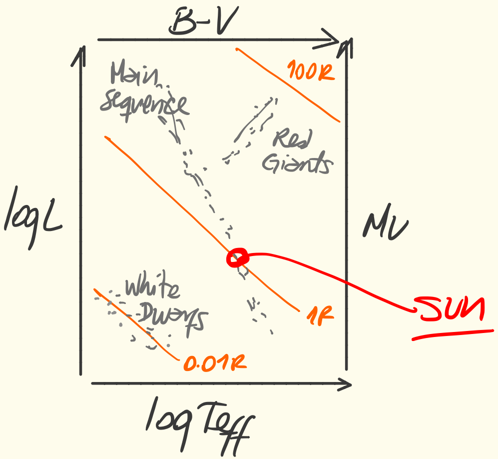
Basic Assumptions:
A. Isolation
B. Uniform initial composition
C. Spherical symmetry

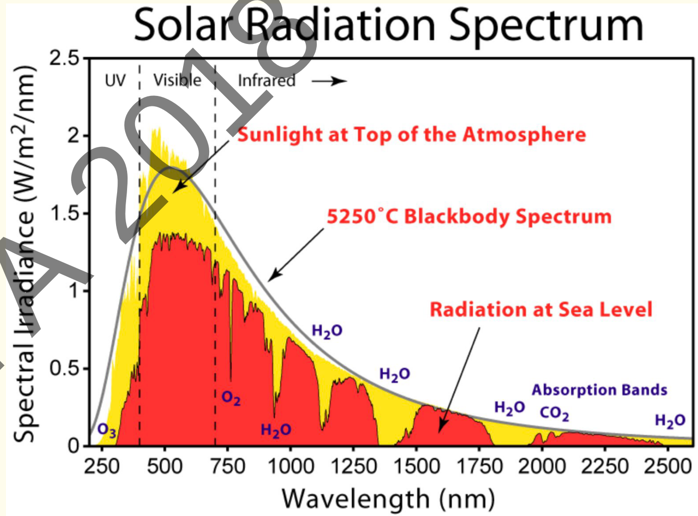


Sun : Temperature$_{eff}$

Measured from spectra continuum

$T_{eff} \sim 5800\,K$





Wien's law

$$\lambda_{max} T \simeq 2900\ \mu m \cdot K$$

T? Units?

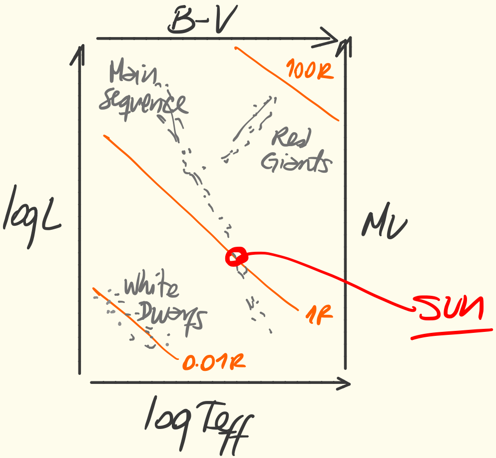
$$L = 4\pi R^2 \sigma T_{eff}^2$$

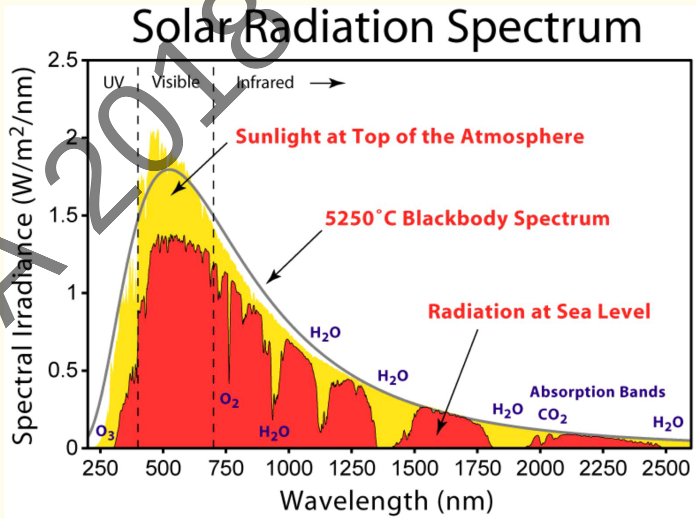
→ Photosphere

Sun : Temperature_{eff}

Measured from spectra continuum

$T_{eff} \sim 5800\,K$ → Sun is not a perfect black body.





Wien's law

$$\lambda_{max} T \simeq 2900\ \mu m \cdot K$$

$$T \simeq \frac{2.9\times10^{6}\,nm\cdot K}{550\,nm}$$

$$T = 5273\,K$$

$$L = 4\pi R^2 \sigma T_{eff}^{2}$$ → Photosphere

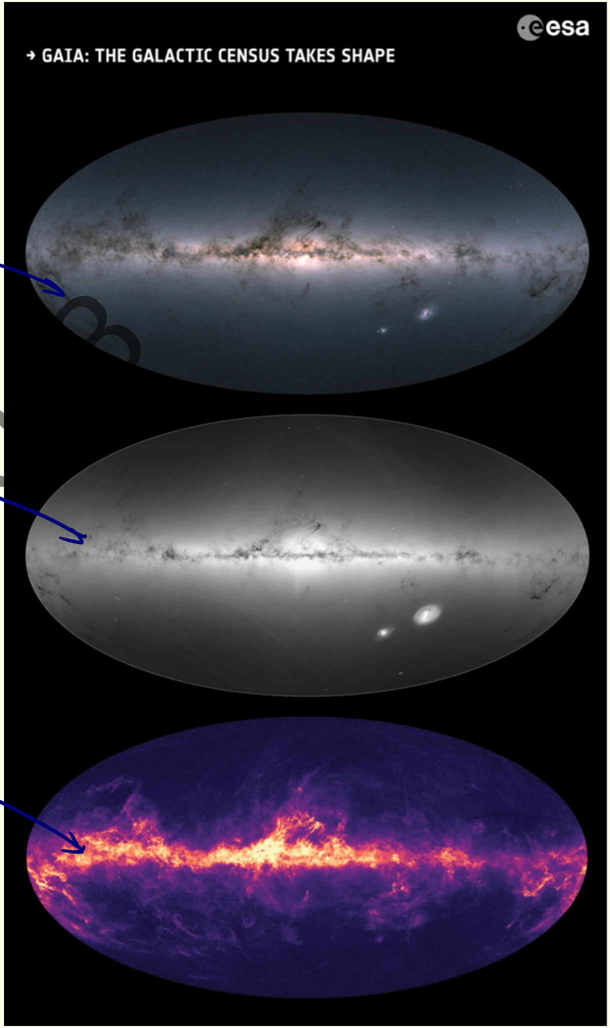
Sun: Distance → Parallaxes

Gaia's two telescopes monitor each of its target stars about 70 times over a five-year period, spinning slowly to sweep the entire celestial sphere. As the telescopes repeatedly measure the position of each celestial object, they detect the combination of the apparent motion caused by the parallax effect and the true motion of the object. By combining the measurements for all objects viewed, it is possible to obtain the parallax and proper motion for each object targeted.

Brightness
Color

Stellar
Density

Dust



$d = 1.5 \times 10^{13}$ cm

Sun : Radius

$R = 7 \times 10^{10}$ cm

$0.01 R_\odot$ to $> 1000 R_\odot$

→ compact stars

tens of km

MEASURING THE SOLAR RADIUS FROM SPACE DURING THE 2003 AND 2006 MERCURY TRANSITS

M. Emilio[1], J. R. Kuhn[2], R. I. Bush[3], and I. F. Scholl[2]

(Dated: Received December 13, 2011; accepted March 5, 2012)
To appear in Astrophysical Journal

Sun : luminosity

Apparent Brightness $I_{obs} = \frac{L}{4\pi d^2}$ → stellar property

$I_{obs\odot} = 1.4 \times 10^6$ erg/s.cm^2 (top atmosphere)

$I_{obs\odot} = 1.4$ kW/m^2 → solar flux over an area

$L_\odot$? Units?

Sun : luminosity

$L_\odot = 3.9 \times 10^{33}$ erg/s

Apparent Brightness $\rightarrow$ $I_{obs} = \dfrac{L}{4\pi d^2}$ $L \rightarrow$ stellar property

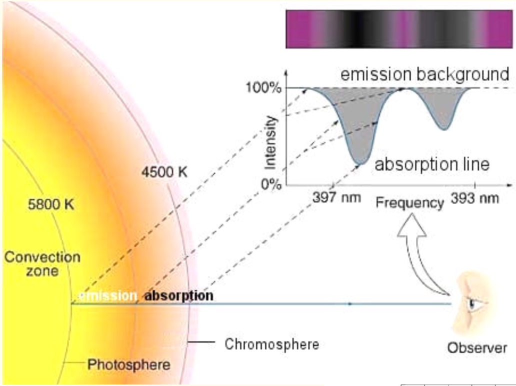
$I_{obs\,\odot} = 1.4 \times 10^6$ erg/s.cm² (top atmosphere)

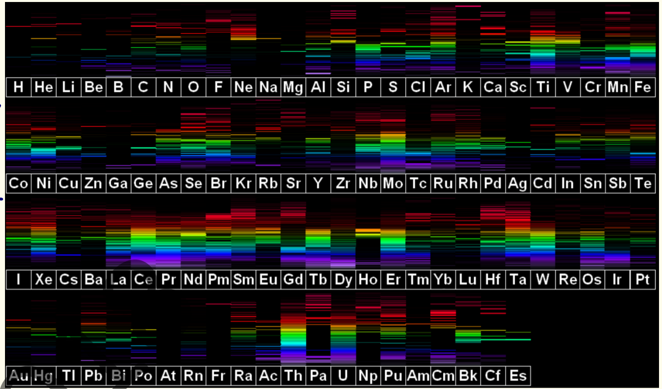
$I_{obs\odot} = 1.4$ kW/m² $\rightarrow$ solar flux over an area

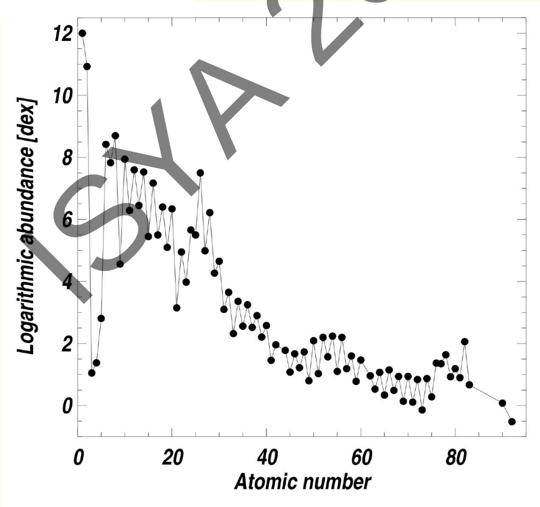
$L_\odot = 4\pi d^2 I_{obs\odot} = 4\pi (1.5 \times 10^{13}\,\text{cm})^2 (1.4 \times 10^6\,\text{erg/s.cm}^2)$

$L_\odot = 3.9 \times 10^{33}$ erg/s $\sim$ $(10^{-5} L_\odot - 10^5 L_\odot)$

Sun: Composition







3D time-dependent hydrodynamical models

$H \equiv X = 0.72$

$He \equiv Y = 0.25$

$\text{Rest} \equiv Z = 0.02$

fractions

Asplund et al. 2009
ARA&A, 47, 481

Sun : Mass

M? Units?

Kepler's law

Newton's law

$$? \leftarrow P^2 = \frac{4\pi^2 a^3}{GM}$$

$a \rightarrow$ 1 AU

$G \rightarrow 6.7 \times 10^{-8}\ cm^3/g/s^2$

Sun : Mass

$M = 2 \times 10^{33}$ g → Kepler's law
Newton's law

$$\underbrace{P^2}_{1\,yr} = \frac{4\pi^2 \overbrace{a^3}^{1\,AU}}{\underbrace{G}_{6.7\times10^{-8}\,cm^3/g/s^2} M}$$

Substellar limit : $0.073 M_\odot$
Massive stars : tens $M_\odot$.
} Range in mass

Sun :

Average Density $\langle\rho\rangle = \dfrac{M}{\frac{4\pi R^3}{3}}$

$\langle\rho\rangle = ???$ Units?

Central Density $\rho_c = \underbrace{c}_{\approx 50 \text{ from models}} \langle\rho\rangle$

$\rho_c = ???$ Units?

If all H → $n_H = \dfrac{\rho_c}{m_H} = \dfrac{?}{1.7\times10^{-24}\,g} = ???$ Units?

Sun: Average Density

$$\langle\rho\rangle = \frac{M}{\frac{4\pi R^3}{3}} = \frac{3}{4\pi}\,\frac{2\times10^{33}\,g}{(7\times10^{10}\,cm)^3}$$

$$\langle\rho\rangle = 1.4\ g/cm^3 \longrightarrow 1\ g/cm^3\ H_2O!$$

Central Density

$$\rho_c = \underbrace{c}_{50 \text{ from models}}\langle\rho\rangle$$

$$\rho_c = 70\ g/cm^3 \longrightarrow \text{Osmium}$$

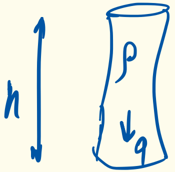
Densest element $22.6\ g/cm^3$

If all H $\rightarrow$ $n_H = \frac{\rho_c}{m_H} = \frac{70\ g/cm^3}{1.7\times10^{-24}\ g} = 4\times10^{25}\ cm^{-3}$

Average interparticle distance $\langle r\rangle \sim n^{-1/3} \longrightarrow 3\times10^{-9}\ cm$

[Bohr radius $= \frac{\hbar^2}{m_e e^2} = 5.3\times10^{-9}\ cm$] — e^- orbital overlap

Sun: Pressure



Pressure at bottom of hydrostatic column

$$P = \langle\rho\rangle\langle g\rangle h$$

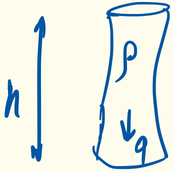
Earth$_{\text{surface}}$ $\rightarrow P = 10^{-3}\ g/cm^3 \cdot 980\ cm/s^2 \cdot 10\ km = 10^6\ \frac{g}{cm \cdot s^2}\ \left(\frac{dyne}{cm^2}\right)$

Earth 10m underwater $\rightarrow P = 1\ g/cm^3 \cdot 980\ cm/s^2 \cdot 10\ m = 10^6\ dyne/cm^2$

Star $\rightarrow P = \langle\rho\rangle\langle g\rangle R = \left(\frac{M}{\frac{4\pi}{3}R^3}\right) \cdot \left(\frac{GM}{R^2}\right) \cdot R \propto \frac{GM^2}{R^4}$

Sun $\rightarrow P \propto \frac{GM^2}{R^4}$ = ??? Units?

Sun : Pressure



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Star $\rightarrow P = \langle \rho \rangle \langle g \rangle R = \left(\frac{M}{\frac{4\pi R^3}{3}} \right) \cdot \left(\frac{GM}{R^2} \right) \cdot R \propto \frac{GM^2}{R^4}$

10^{10} atmospheres!

Sun $\rightarrow P \propto \frac{GM^2}{R^4} = \frac{(6.7 \times 10^{-8}\ cm^3/g \cdot s^2)(2 \times 10^{33}\ g)^2}{(7 \times 10^{10}\ cm)^4} = 10^{16}\ \frac{dyne}{cm^2}$

Central pressure $\sim 2 \times 10^{17}\ dyne/cm^2 = P_c$

Class 1 Review:

1. A star fulfills two conditions along a time range:
   A. Bound by self-gravity
   B. Radiates energy supplied by an internal source.

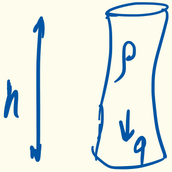
2. We locate the stars in a plane of:
   Luminosity vs. T_{eff} ⟷ Color vs. Magnitude
   Inferred parameters — observables

3.

The Sun:

- $T_{eff} \sim 5800K \rightarrow 5527\,°C$ (wash (lava) at 2000°C)
- $d = 1.5\times10^{13}$ cm $\longrightarrow$ 1 AU
- $R = 7\times10^{10}$ cm $\longrightarrow$ 109 time Earth's diameter
- $L_\odot = 3.9\times10^{33}$ erg/s $\longrightarrow$ ($10^{-5} L_\odot - 10^{5} L_\odot$)
- $H \equiv X = 0.72$, $He \equiv Y = 0.25$, Rest $\equiv Z = 0.02 \longrightarrow$ G2V
- $M = 2\times10^{33}$ g $\longrightarrow$ substellar limit $\sim 0.073\, M_\odot \sim 73\, M_{Jup}$
- $\langle\rho\rangle = 1.4$ g/cm³ $\longrightarrow$ water 1 g/cm³
- $\rho_c = 70$ g/cm³ $\longrightarrow$ Osmium 22.6 g/cm³

Sun : Pressure



Pressure at bottom of hydrostatic column

$$P = \langle \rho \rangle \langle g \rangle h$$

Earth$_{\text{surface}}$ $\rightarrow P = 10^{-3}\ \mathrm{g/cm^3} \cdot 980\ \mathrm{cm/s^2} \cdot 10\ \mathrm{km} = 10^{6}\ \frac{\mathrm{g}}{\mathrm{cm \cdot s^2}} \left(\frac{\mathrm{dyne}}{\mathrm{cm^2}}\right)$

Earth 10 m underwater $\rightarrow P = 1\ \mathrm{g/cm^3} \cdot 980\ \mathrm{cm/s^2} \cdot 10\ \mathrm{m} = 10^{6}\ \mathrm{dyne/cm^2}$

Star $\rightarrow P = \langle \rho \rangle \langle g \rangle R = \left(\frac{M}{\frac{4\pi}{3} R^3}\right) \cdot \left(\frac{GM}{R^2}\right) \cdot R \propto \frac{GM^2}{R^4}$

Sun $\rightarrow P \propto \frac{GM^2}{R^4} = \frac{(6.7\times10^{-8}\ \mathrm{cm^3/g \cdot s^2})(2\times10^{33}\ \mathrm{g})^2}{(7\times10^{10}\ \mathrm{cm})^4} = 10^{16}\ \frac{\mathrm{dyne}}{\mathrm{cm^2}}$

10^{10} atmospheres!

Central pressure $\sim \underline{2\times10^{17}}\ \mathrm{dyne/cm^2} = P_c$

Sun: Central temperature.

Eddington: high internal temp $\rightarrow$ ionized material $\rightarrow$ $\gamma + e^-$ $\rightarrow$ ideal gas

1.38×10^{-16} erg/K

$$P_c = n k T_c$$

2×10^{17} dyne/cm^2

4×10^{25} cm^{-3}

??? Units?

Sun: Non-relativistic ideal gas $\leftarrow$ $T_{rel} \sim 2 \times 10^{9}$ K

$$\lambda_{max} T = 2900\ \mu m \cdot K \longrightarrow \lambda_{max} = ???$$ Units?

Sun : Central temperature.

Eddington: high internal temp → ionized material →

$\gamma + e^- \rightarrow$ ideal gas

1.38×10^{-16} erg/K

$P_c = nkT_c$

2×10^{17} dyne/cm^2

4×10^{25} cm^{-3}

$T_c = 4 \times 10^7 K$

Models $T_c = 1.5 \times 10^7 K$

Sun: Non-relativistic ideal gas ← $T_{rel} \sim 2 \times 10^9 K$

$\lambda_{max} T \simeq 2900 \, \mu m \cdot K \longrightarrow \lambda_{max} = ???$ Units?

Sun: Central temperature.

Eddington: high internal temp → ionized material → $\gamma + e^- \rightarrow$ ideal gas

1.38×10^{-16} erg/K

$P_c = nkT_c$

2×10^{17} dyne/cm²

4×10^{25} cm⁻³

$T_c = 4 \times 10^7$ K

Models $T_c = 1.5 \times 10^7$ K

Sun: Non-relativistic ideal gas ← $T_{rel} \sim 2 \times 10^9$ K

$$\lambda_{max} T \simeq 2900\ \mu m \cdot K \longrightarrow \lambda_{max} = \frac{2900\ \mu m \cdot K}{1.5 \times 10^7 K} = 1.9\ \text{Å}$$

$\boxed{\text{central}}$

wavelength of thermal radiation

X-rays

Sun: Age

Isotope dating on meteorites $4.6 \times 10^9 yr$

Impossibility of a long term observation

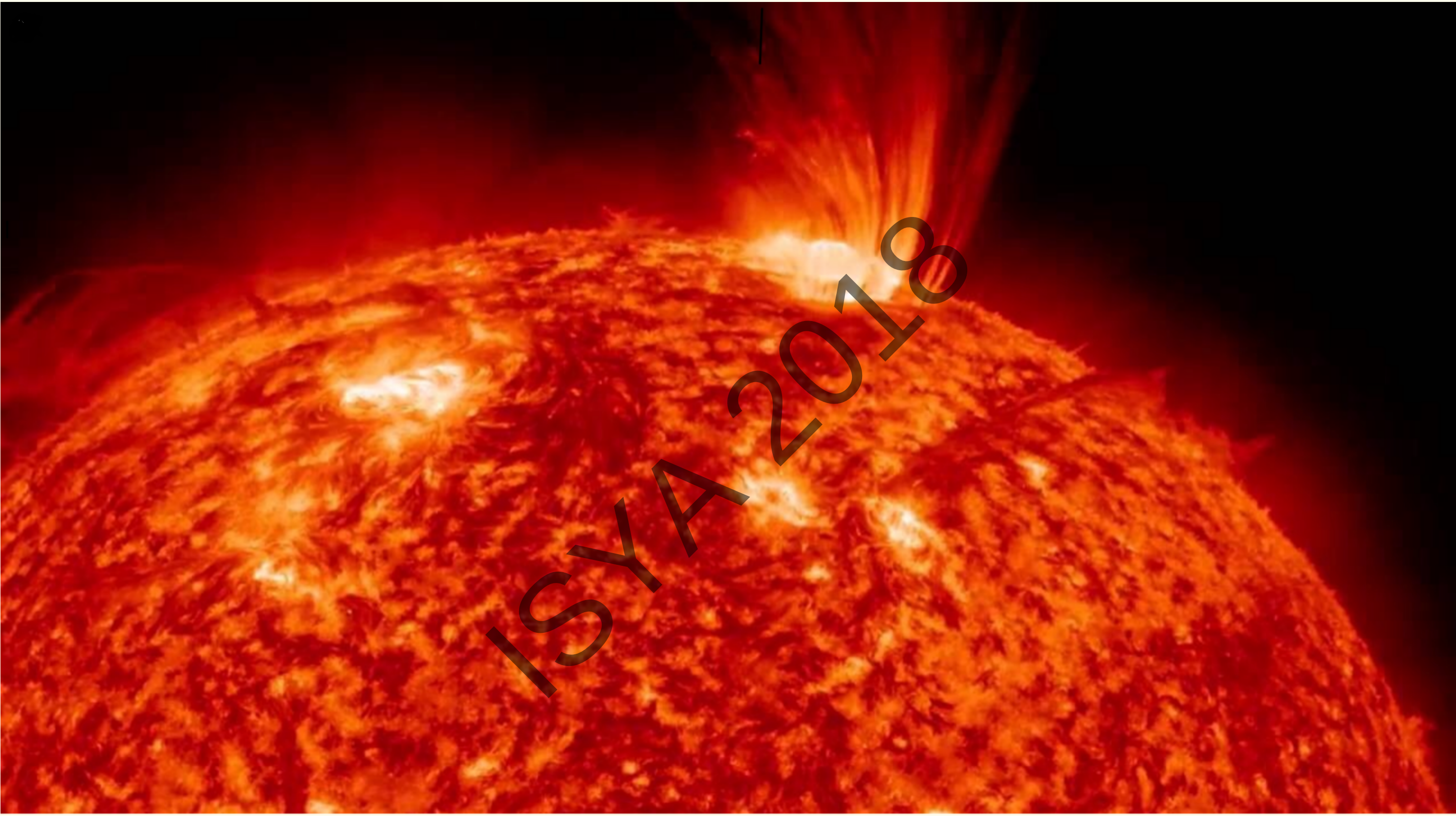
↓

Observation of different sources at different evolutionary steps

↓

Not individual stars → General model

Since Δt is small → Theoretical models



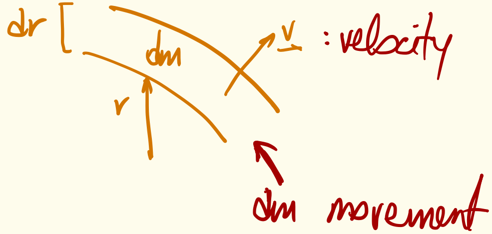
Equations of stellar structure

Radiant } H + He (mainly)
gaseous sphere

Structure of stars with mass M → uniquely determined t, ρ, T, X at each "point" (r or m)

Stars:

$\boxed{1}$ Mass continuity



Interplay!

$$\underbrace{\frac{\partial m}{\partial r} = 4\pi\rho r^2}_{\text{Static}} \sim \cdot \frac{dt}{dt} \rightarrow \underbrace{\partial m = -4\pi r^2 \rho \underline{v} \partial t}_{\text{Non-static}}$$

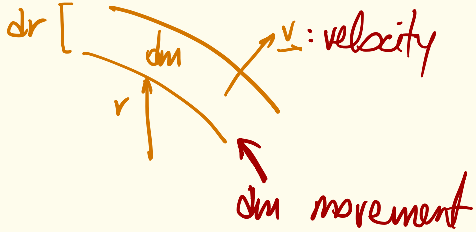
Mass flux change at a given r.

Deriving ∂t + spheric symmetry

$$\frac{\partial \rho}{\partial t} = -\mathrm{div}(\rho \underline{v})$$

Stars:

[1] Mass continuity



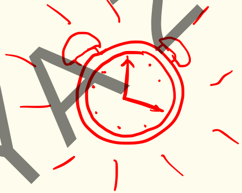
Interplay!

$$\underbrace{\frac{\partial m}{\partial r} = 4\pi \rho r^2}_{\text{Static}} \sim \cdot \frac{\partial t}{\partial t} \longrightarrow \underbrace{\partial m = -4\pi r^2 \rho \underline{v}\, \partial t}_{\text{Non-static}}$$

Mass flux change at a given r.

Deriving ∂t + spheric symmetry

$$\frac{\partial \rho}{\partial t} = -\operatorname{div}(\rho \underline{v})$$



Continuity equation

↓

Transport of a quantity $\longrightarrow$

In fluids

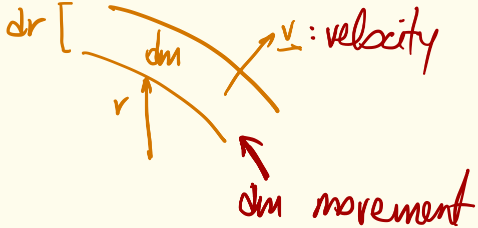
$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \underline{v}) = 0$$

$\longrightarrow$ conserved quantity

No sources or sinks

Stars:

$\boxed{1}$ Mass continuity



Interplay!

$$\underbrace{\frac{\partial m}{\partial r} = 4\pi \rho r^2}_{\text{Static}} \sim \cdot \frac{dt}{dt} \longrightarrow \underbrace{\partial m = -4\pi r^2 \rho \underline{v}\, \partial t}_{\text{Non-static}}$$

Mass flux change at a given r.

Deriving at + spheric symmetry

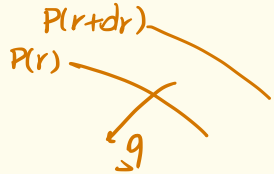
$$\frac{\partial \rho}{\partial t} = -\mathrm{div}(\rho \underline{v})$$

Fluid continuity equation!!!
↳ transport of m!

So in a star m behaves like a fluid and it is a <u>conserved</u> quantity

Stars:

$\boxed{Z}$ Movement



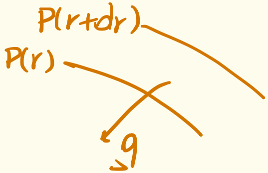
Force balance,

? Forces?
Direction?

What happens if there is no movement?

Stars:

$\boxed{2}$ Movement



Force balance,

Gas surrounding

$$\rho \underline{a} = -\nabla \underline{P} + \underline{F}_{grav}$$

Fluid element

Changes in pressure

Gravitational potential

$$-\rho \frac{Gm(r,t)}{r^2} = -\rho \frac{\partial \phi}{\partial r}$$

$$\rho \underline{a} = -\frac{\partial P}{\partial r} - \rho \frac{\partial \phi}{\partial r}$$

Radial gradient pressure vs. gravitational potential

$\cancel{\rho \underline{a}}^{\,0}$ if hydrostatic equilibrium (No movement)

In terms of mass/radius elements,

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2}$$

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$$

If there is no longer an equilibrium ... Characteristic Times

Ⓐ No pressure. Collapse.

. Free-fall time

$$\frac{\partial P}{\partial r} = 0 \longrightarrow a = \frac{GM}{r^2} = \frac{r}{\tau_{ff}^2} \qquad \tau_{ff} = \sqrt{\frac{R^3}{GM}} \qquad \odot = ?\ \text{minutes?}$$

(average value)

Ⓑ No gravity. Explosion.

Explosion time

$$\frac{GM}{r^2} = 0 \longrightarrow a = -\frac{1}{\rho}\frac{\partial P}{\partial r} = \frac{r}{\tau_{exp}^2} \qquad \tau_{exp} \approx R\left(\frac{\rho}{P}\right)^{1/2} \qquad \odot \approx ?\ \text{minutes?}$$

If there is no longer an equilibrium ... Characteristic Times

Ⓐ No pressure. Collapse.

$$\frac{\partial P}{\partial r} = 0 \longrightarrow a = \frac{GM}{r^2} = \frac{r}{\tau_{ff}^2}$$

Free-fall time

$$\tau_{ff} = \sqrt{\frac{R^3}{GM}}$$

RG = 18 days
☉ = 54 minutes
WD = 4 seconds

Ⓑ No gravity. Explosion.

$$\frac{GM}{r^2} = 0 \longrightarrow a = -\frac{1}{\rho}\frac{\partial P}{\partial r} = \frac{r}{\tau_{exp}^2}$$

Explosion time

$$\tau_{exp} \approx R\left(\frac{\rho}{P}\right)^{1/2}$$

☉ ≈ 14 minutes

If $\tau_{ff} \simeq \tau_{exp} \equiv \tau_{dynamical}$ → Time scale of reaction to a perturb.

Dynamical time scale

Short scales! → Fast reaction!

Piston model → Time to adjust

Stars:

$\boxed{3}$ Energy

Energy → Gravitational Contraction + Nuclear Reactions

A. Gravitational contraction → & hydrostatic equilibrium → Virial Theorem

$$-\frac{\partial P}{\partial r} = \frac{GM}{r^2}\rho \longrightarrow -3\int_0^M \frac{P}{\rho}\,dm = E_{grav}$$

If the star contracts quasi-statically (slowly), the pressure increases given that $|E_{grav}|$ increases and V decreases.

Stars:

$\boxed{3}$ Energy

Energy → Gravitational Contraction + Nuclear Reactions

Ⓐ Gravitational contraction → @ hydrostatic equilibrium → Virial Theorem + Ideal gas

$$-\frac{\partial P}{\partial r} = \frac{GM}{r^2}\rho \longrightarrow E_{int} = -\frac{1}{2} E_{grav}$$

Gravity – E internal relation

A tighly bound system → Higher internal energy → Hotter!

In the sun @ constant density

$$E_{grav} = -\frac{3}{5}\frac{GM_\odot^2}{R_\odot}$$

In a ~~lifetime~~ @ constant luminosity

$$\tau = \frac{E_{int}}{L_\odot} \approx ???$$

Surprised?! yr!

Stars:

$\boxed{3}$ Energy

Energy → Gravitational Contraction + Nuclear Reactions

Ⓐ Gravitational contraction → @ hydrostatic equilibrium → Virial Theorem + Ideal gas

$$-\frac{\partial P}{\partial r} = \frac{GM}{r^2}\rho \longrightarrow E_{int} = -\frac{1}{2} E_{grav}$$

In the sun @ constant density

$$E_{grav} = -\frac{3}{5}\frac{GM_\odot^2}{R_\odot}$$

In a lifetime @ constant luminosity

$$\tau = \frac{E_{int}}{L_\odot} \approx 10^7 \text{yr}$$

Or thermal τ_{KH}

Lifetime of contraction, without nuclear reactions

Gravitational contraction is not enough to sustain the Sun power output along its age!

Isotope dating age for the Solar system 4.6×10^9 yr

scale of changes in structure

Stars:

$\boxed{3}$ Energy

Energy → Gravitational Contraction + Nuclear Reactions

(B) Nuclear Reactions — for each H used to create He (4 are needed)

↓

Release energy: 6.3×10^{18} erg/gr

If all the available H is burned?

? star $= 1 M_\odot$ made entirely by H → Energy released

$1 M_\odot * 6.3 \times 10^{18}$ erg/gr

12×10^{52} erg !!!

For how long this is sustainaible?

$$\tau_{nuc} = \frac{12 \times 10^{52}\,\text{erg}}{L_\odot} \sim ??\ \text{yr.}$$

Why?

scale of changes in composition

Stars:

$\boxed{3}$ Energy

Energy → Gravitational Contraction + Nuclear Reactions

B. Nuclear Reactions: for each H used to create He (4 are needed)

↓

Release energy: 6.3×10^{18} erg/gr

If all the available H is burned?

a star $= 1M_\odot$ made entirely by H → Energy released: $1M_\odot * 6.3 \times 10^{18}$ erg/gr

12×10^{52} erg!!!

For how long this is sustainaible?

$$\tau_{nuc} = \frac{12 \times 10^{52}\,\text{erg}}{L_\odot} \sim 10^{11}\,\text{yr}$$

scale of changes in composition

} Enough time. (Idealized)

$\tau_{nuc} \gg \tau_{KH} \gg \tau_{dynamical}$

Timescale for abundance changes $\gg$ Timescale for structure changes $\gg$ Timescale for star to adjust

Stars:
$\boxed{3}$ Energy

Energy → Gravitational Contraction + Nuclear Reactions

Additional sources of energy change,
- Material exchange
- Irradiated luminosity
- Irradiated luminosity from neutrinos

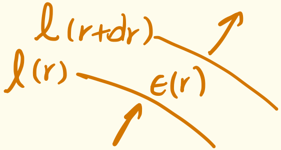
$$\frac{d}{dt}\left(E_{int} + E_{grav}\right) = -L - L_{\nu} + G_{nuclear}$$

If thermal equilibrium → constant energy

$$L \simeq G_{nuclear}$$

Stars:

$\boxed{3}$ Energy generation



Lets define an energy generation rate $\quad \epsilon \equiv \dfrac{\text{energy generated}}{\text{time} \cdot \text{mass}} \quad (\text{erg/s/g})$

In a shell $\quad \ell(r+dr) - \ell(r) = \epsilon\, dm = \epsilon\, 4\pi r^2 \rho(r)\, dr$

$$\frac{d\ell}{dr} = 4\pi r^2 \rho \epsilon \quad \text{or} \quad \frac{d\ell}{dm} = \epsilon$$

Nuclear energy production can be described as,

$$\epsilon = \epsilon_0 \rho^{\lambda} T^{\nu} \longrightarrow \frac{d\ell}{dr} = 4\pi r^2 \epsilon_0 \rho^{\lambda+1} T^{\nu}$$

$(\lambda, \nu) \approx (1, 4)$ H→He
$(1, 15)$ H→He CNO
$(2, 40)$ He→C

Energy production as function of r

→ How it is transported?

Stars:

$\boxed{4}$ Energy transfer (heat transfer)

A. Radiation: energy transfer in an opaque medium as a random walk of photons

Path lenght for the photon motion $\quad l_p = \frac{1}{\kappa_{rad}\,\rho}$

opacity (cm^2/g)

density (g/cm^3)

less opacity better transfer

In Sun, κ due to thompson scattering

$l \approx 2cm$

Ionized medium

small deviations per photon compared with stellar inhomogeneities

Photons are absorbed and reemited several times before finding a medium inhomogeneity. $\approx$ Black body

If it is a perfect black body $\longrightarrow$ $F_{rad} = 0$

Stars:

$\boxed{4}$ Energy transfer (heat transfer)

But in general,

$$F_{rad} = \frac{-16\sigma T^3}{3\rho \kappa_{rad}} \operatorname{grad} T \longrightarrow \frac{dT}{dr} = \frac{-3\kappa_0}{16\pi a c} \rho^{n+1} T^{-(s+3)} l$$

$$\kappa_{rad} \equiv \kappa_0 \rho^n T^{-s}$$

Two main opacity origins (high T, few bound electrons, all ionized)

Dispersion of photons due to free electrons

Free-free absorption of a photon by a free-electron

$$\kappa^{th} = 0.2(1+X)\,\frac{cm^2}{g} \qquad \kappa^{ff} \approx 3.8\times10^{22}(1+X)\rho T^{-7/2}\,\frac{cm^2}{g}$$

(Thompson scattering) X Hydrogen abundance (Inverse Brehmstrahlung)

Sun???

Stars:

$\boxed{4}$ Energy transfer (heat transfer)

But in general,

$$F_{rad} = \frac{-16\sigma T^3}{3\rho K_{rad}} \operatorname{grad} T \longrightarrow \frac{dT}{dr} = \frac{-3K_0}{16\pi a c} \rho^{n+1} T^{-(s+3)} l$$

$$K_{rad} \equiv K_0 \rho^n T^{-s}$$

Two main opacity origins (high T, few bound electrons, all ionized)

Dispersion of photons due to free electrons

Free-free absorption of a photon by a free-electron

$$K^{th} = 0.2(1+X) \frac{cm^2}{g}$$

$$K^{ff} \approx 3.8\times10^{22}(1+X)\rho T^{-7/2} \frac{cm^2}{g}$$

X Hydrogen abundance

(Thompson scattering)

(Inverse Brehmstrahlung)

Sun, $\rho = 1.4 \frac{gr}{cm^3}$, $T \sim 10^7 K \longrightarrow K^{th} = 0.4 \frac{cm^2}{g}$ vs. $K^{ff} \approx 0.03 \frac{cm^2}{g}$

Stars:

$\boxed{4}$ Energy transfer (heat transfer)

B. Conduction:

$$K_{cond} \longrightarrow F_{cond} = \frac{-16\sigma T^3}{3\rho K_{cond}} \nabla T$$

Relevant at high densities associated with degenerated matter.

C. Convection:

Heat transport with the gas → Collective (bulk) motions of gas particles

A perfect convection process → $P \propto \rho^{3/5}$ polytrope relation.

More latter...

Star evolution → quasi-static process
(slow)

(adjust internal structure to keep dynamical balance]

Slow composition changes → hydrostatic equilibrium
thermal equilibrium
(constant energy)

Space variables, r or m:

$$\frac{dm}{dr} = 4\pi r^2 \rho \qquad \frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}$$ Continuity Equation

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2} \qquad \frac{dP}{dm} = -\frac{Gm}{4\pi r^2}$$ Hydrostatic Equilibrium

$$\frac{dl}{dr} = 4\pi r^2 \rho \epsilon \qquad \frac{dl}{dm} = \epsilon$$ Thermal Equilibrium

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{l}{4\pi r^2} \qquad \frac{dT}{dm} = -\frac{3}{4ac}\frac{\kappa}{T^3}\frac{l}{(4\pi r^2)^2}$$ Energy Transfer (radiative)

(keep in mind conduction + convection)

Observational capsule:

1. If the star can not recover from a dynamical process

It can not restore: ??

So it ___?___ or ___?___

And we observe ?

2. If we observe rapid changes in the star

we are seeing ___?___ process

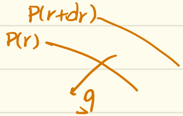
Oscillations with periods of few tens of seconds indicate a compact star, such as ___?___

Class 2 Review:

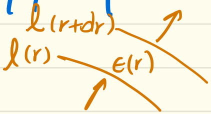
Equations of stellar structure

$\boxed{1}$ Mass continuity: m behaves like a fluid, following a continuity equation of a conserved quantity.

$\boxed{2}$ Movement: Radial gradient pressure vs. gravitational potential. Hidrostatic equilibrium.



$\boxed{3}$ Energy generation: gravitational contraction
+
nuclear reaction
thermal equilibrium



$\boxed{4}$ Energy transport: radiation, conduction, convection
Opacity

Class 2 Review:

$$\tau_{nuc} \gg \tau_{KH} \gg \tau_{dynamical}$$

Timescale for abundance changes	$\gg$	Timescale for structure changes	$\gg$	Timescale for star to adjust
↓		↓		↓
$\boxed{3}$ Energy Nuclear reaction		$\boxed{3}$ Energy Gravitational contraction		$\boxed{1}$ Movement Equation

Until now . . .

Sun + 4 main equations
- mass continuity
- mass movement
- Energy generation
- Energy transfer

& ideal gas
classical physics

A more robust approach,

Distribution function of material → Distribution of particles in position-momentum space → Statistical mechanics + isotropy

Until now ...

A more robust approach,

Distribution function of material (2) → Distribution of particles in position-momentum space (1) → Statistical mechanics + isotropy

(1) Equation of state stellar gas

$P = f(\rho, T, X_i)$

(2) Distribution function

$f(\underline{x}, \underline{p})$

$$n = \int f(\underline{x}, \underline{p})\, d^3p$$ Number density

$$P = \frac{1}{3}\int f(\underline{x}, \underline{p})\, p v\, d^3p$$ Pressure

$$u = \int f(\underline{x}, \underline{p})\, E(p)\, d^3p$$ Energy density

① Equation of state
stellar gas
$P = f(\rho, T, X_i)$

Extreme physical conditions
↳ Quantum-mechanic effects (e^- degeneracy)
↳ Relativistic effects

Non-ideal effects: Outer layers with $T < 10^6$ K not fully ionized.

Electrostatic interactions among ions and electrons.

① Equation of state

stellar gas

$P = f(\rho, T, X_i)$

Classical limit:

Non-degenerated case

$P = nkT$

But this is a gas of identical particles of mass m.

↓

Mixture of gases

$P_e = \frac{R}{\mu_e} \rho T$

$P_{ion} = \frac{R}{\mu_{ion}} \rho T$

universal gas constant

$P_{gas} = \frac{R}{\mu} \rho T$

mean molecular weight (density of ions and electrons)

Atomic mass per particle

$\mu = \frac{\sum_j n_j A_j}{\sum_j n_j + n_e}$

Relativistic / Non-relativistic

① Equation of state stellar gas

$P = f(\rho, T, X_i)$

Quantum-mechanical limit:

Degenerated case

In quantum mechanics: $\Delta x \Delta p \geq h$ $\xrightarrow{3D}$ $\Delta V \Delta^3 p \geq h^3$

limited accuracy location + momentum

number of quantum states (6D)

Occupation of the quantum states → Bosons (photons): No restrictions

→ Fermions (e^- or nucleons): Pauli exclusion principle

No two particles can occupy the same quantum state

e^- exert extra pressure!!! → Degeneracy pressure

① Equation of state
stellar gas

$$P = f(\rho, T, X_i)$$

Quantum-mechanical limit $(T \to 0)$

Degenerated case

μ_e: Mean molecular weight per free electron

$$P_e = 10^{13} \left(\frac{\rho}{\mu_e}\right)^{5/3} \left(\frac{erg}{cm^3}\right)$$

Non-relativistic

$$P_e = 1.2 \times 10^{15} \left(\frac{\rho}{\mu_e}\right)^{4/3} \left(\frac{erg}{cm^3}\right)$$

Relativistic

Density $\uparrow$
Momentum $\uparrow$
e^- velocity $\uparrow$

Independent of T!

② Distribution function

$$f(\underline{x}, \underline{p})$$

Non-degenerated case

$$f^{3D}_{MB} = \frac{n_e}{(2\pi m_e kT)^{3/2}} \exp\left(\frac{-\underline{p}^2}{2m_e kT}\right)$$

Maxwell-Boltzmann

Degenerated case

$$f^{3D}_{FD} = \frac{2}{h^3} \frac{1}{1+\exp\left(\frac{\underline{p}^2}{2mkT} - \frac{\mu}{kT}\right)}$$

Fermi-Dirac

$M > M_\odot$

$$\frac{n_e}{T^{3/2}} \leq 0.1\, H_{FD}$$

$$\frac{\mu}{kT} < -2$$

$$\frac{\rho}{T_4^{3/2}} \leq 1.8\times 10^{-3} \mu_e$$

$M < M_\odot$

$$\frac{n_e}{T^{3/2}} \geq 10\, H_{FD}$$

$$\frac{\mu}{kT} \geq 10$$

$$\frac{\rho}{T_4^{3/2}} \geq 0.2\, \mu_e$$

Where $H_{FD} = 1.09\times 10^{16} (K^{-3/2} cm^{-3})$, $T_4 = \frac{T(K)}{10^4 K}$

Presure of a mixture of gas and radiation

$$P = P_{rad} + P_{gas}$$

$$P = \frac{1}{3} a T^4 + P_{gas}$$

$$P = \frac{1}{3} a T^4 + P_{ion} + P_e$$

In a mixture of gases the ionization is partial
Inter-dependency of species density.

Classical

Quantum (Degenerated)

Non-relativistic

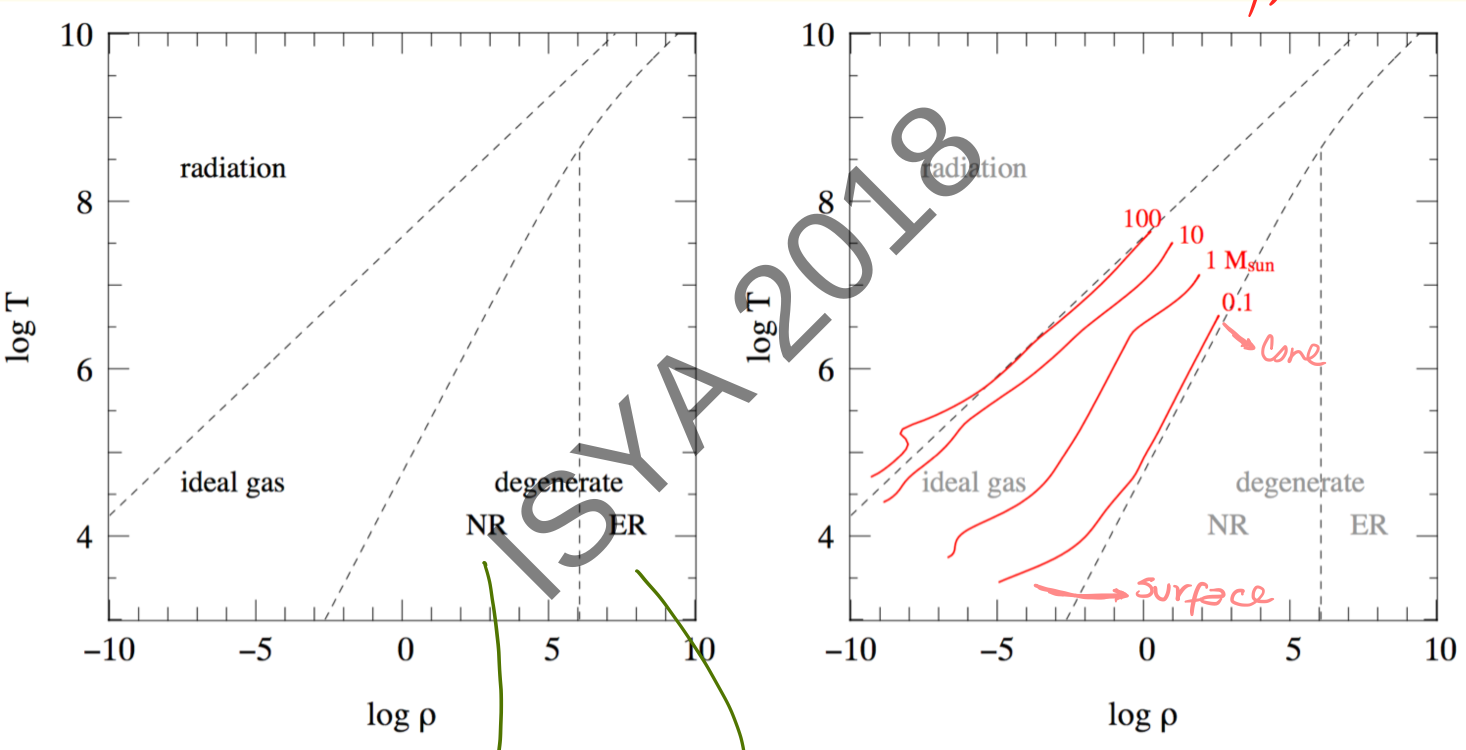
Relativistic

a: radiation constant

The $\log T$, $\log \rho$ plane:

zero-age (homogeneous) structure models

(Solar Metallicity)



Non-relativistic Extremely relativistic

Electron Degeneracy

Source with a high core density and starts contraction

Team up!
15 min

$\boxed{A}$ What happens with:

1. core electrons
2. Pressure
3. Temperature
4. Hydrostatic equilibrium

$\boxed{B}$ Why the source transforms from this:

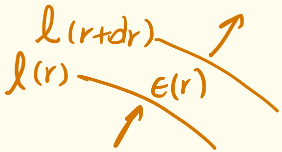
$$R \propto M^{-1/3}$$

To this

$$M = \text{constant}$$

Until now . . .

$$\epsilon \equiv \frac{\text{energy generated}}{\text{time} \cdot \text{mass}} \quad (\text{erg/s/g})$$



A more robust approach,

Detailed element conversion $\longrightarrow$ Reaction chains

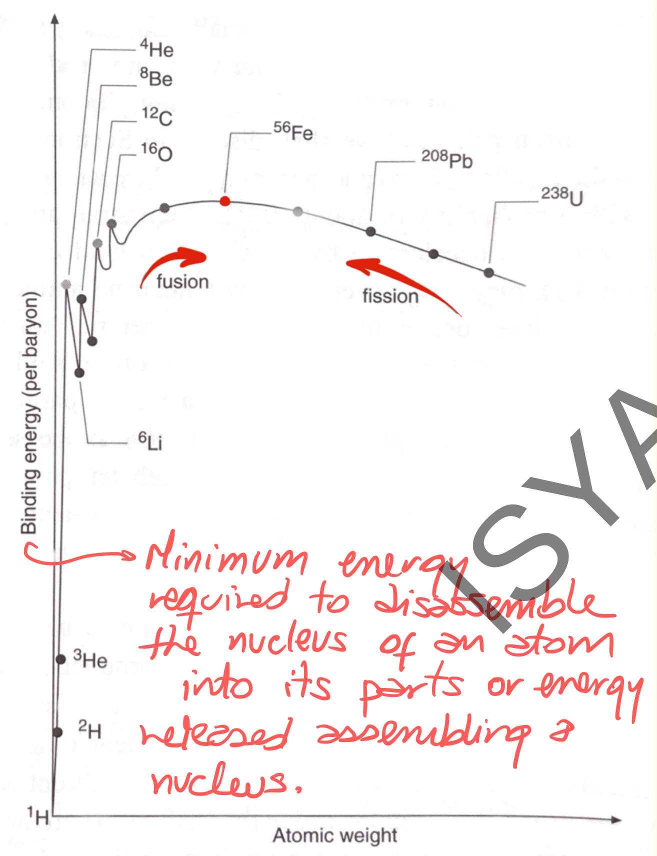
Change composition $\rightarrow$ transmutation of elements

1. H $\longrightarrow$ He — via PP cycle ($T < 1.4 \times 10^7$ K)
   — via CNO cycle ($T > 1.4 \times 10^7$ K)

2. He $\longrightarrow$ ^{12}C, ^{16}O, ^{20}Ne

3. Final cycles

Until ^{56}Fe the reactions produce energy
→ end of fusion processes



→ Minimum energy required to disassemble the nucleus of an atom into its parts or energy released assembling a nucleus.

$^{2}H \rightarrow {}^{56}Fe$: 8.8 MeV
$^{2}H \rightarrow {}^{4}He$: 7.0 MeV !!!

Energy is produced by fusion of light nuclei into heavier ones → up to iron

Energy is produced by fission of heavy nuclei into lighter ones → down to iron.

① H → He — via PP cycle ($T < 1.4 \times 10^7$ K)

1.4×10^{10} y .... $^1H + {}^1H \rightarrow {}^2H + e^+ + \nu$
1.44 MeV

positron
neutrino
deuterium

6s .... $^2H + {}^1H \rightarrow {}^3He + \gamma$
5.49 MeV

photon

69% | 31%

10^6 y ... $^3He + {}^3He \rightarrow {}^4He + {}^1H + {}^1H$
12.85 MeV

pp1

$^3He + {}^4He \rightarrow {}^7Be + \gamma$

99.7% | 0.3%

$^7Be + e^- \rightarrow {}^7Li + \nu$

$^7Li + {}^1H \rightarrow {}^4He + {}^4He$

pp2

$^7Be + {}^1H \rightarrow {}^8B + \gamma$

$^8B \rightarrow {}^8Be + e^+ + \gamma$

$^8Be \rightarrow {}^4He + {}^4He$

pp3

Energy released + Stability

ppX → Temperature

- The produced γ are absorbed by the stellar plasma increasing the system energy.
- The produced ν scape from the system (only relevant in pp3)

① H $\rightarrow$ He — via CNO cycle ($T > 1.4 \times 10^7$ K)

10^6 y . . . . $^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma$

14 min . . . . $^{13}N \rightarrow {}^{13}C + e^- + \nu$

10^5 y . . . . $^{13}C + {}^{1}H \rightarrow {}^{14}N + \gamma$

10^8 y . . . . $^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma$

82 s . . . . $^{15}O \rightarrow {}^{15}N + e^+ + \nu$

99.96%   0.04%

10^4 y . . . . . . $^{15}N + {}^{1}H \rightarrow {}^{12}C + {}^{4}He$

$^{15}N + {}^{1}H \rightarrow {}^{16}O + \gamma$

$^{16}O + {}^{1}H \rightarrow {}^{17}F + \gamma$

$^{17}F \rightarrow {}^{17}O + e^+ + \nu$

$^{17}O + {}^{1}H \rightarrow {}^{14}N + {}^{4}He$

Catalyst reaction!

Highly sensitive at changes in temperature!

Cycle comparison,

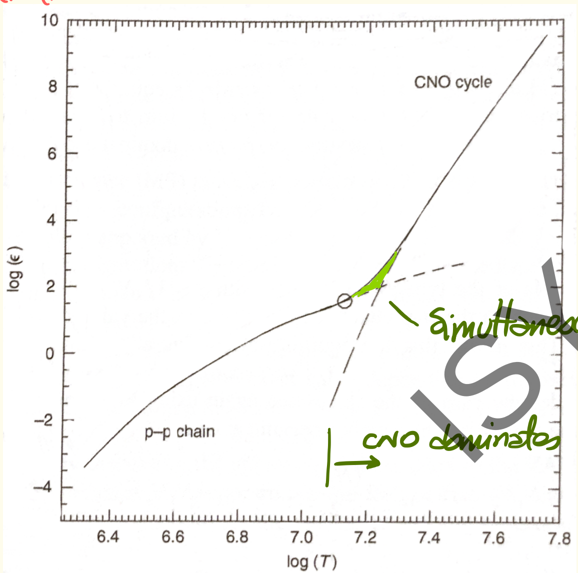
Energy release $\epsilon_{pp} \propto \rho T^4$ Weak temperature sensitivity

Decay based

Energy release $\epsilon_{CNO} \propto \rho T^{16}$ Extremely sensitive to temperature

Capture based

Energy generation



If T↓, cycle PP relevant, $M < M_\odot$

If T↑, cycle CNO relevant, $M > M_\odot$

If T↑, there are more reactions per second, the H is exhausted before → shorter live.

Note efficiency: less 1% of initial rest mass turns into energy by the fusion H → He → Inefficient. But large reservoirs H!

② He $\rightarrow$ ^{12}C, ^{16}O, ^{20}Ne

$3\,^4He$ $\quad$ ^4He $\quad$ $^{12}C + ^{12}C$

- It requires $T > 10^8 K$ $\rightarrow$ not in the main sequence.
- It is highly sensitive to temperature changes.
- Energy release $\epsilon_{3\alpha} \propto \rho^2 T^{40}$
- High production of light particles $\rightarrow$ different isotopes.

③ Final cycles

- It requires $T > 10^9 K$

$$^{12}C + ^{12}C \left\langle \begin{array}{l} ^{20}Ne + ^4He \\ ^{20}Na + ^1H \end{array} \right.$$

$$^{16}O + ^{16}O \left\langle \begin{array}{l} ^{28}Si + ^4He \\ ^{31}P + ^1H \\ ^{31}S + n \end{array} \right.$$

When $T > 3 \times 10^9 K$ $\quad ^{28}Si \rightarrow \cdots \rightarrow \left\langle \begin{array}{l} ^{56}Fe \\ ^{56}Ni \end{array} \right.$ Peak of production

When $T > 7 \times 10^9 K$ $\quad ^{56}Fe \rightarrow 13\,^4He + 4n$ Process reversal

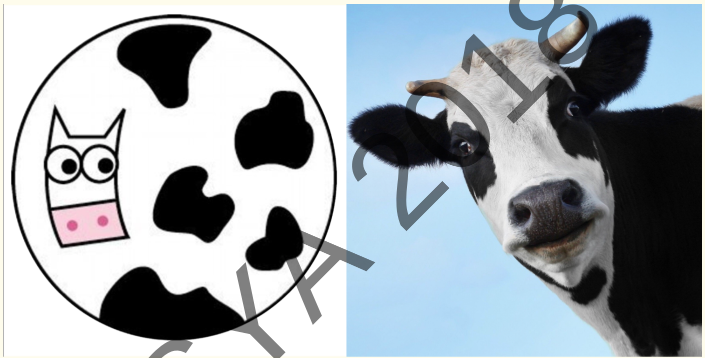
Energetic γ: photodisintegration (not photoionization)

Distribution function of material → Distribution of particles in position-momentum space → Equation of state

Classical

Degenerated

Non-relativistic   Relativistic



Nuclear energy generation → Detailed element conversion → Reaction chains (Temperature Regimes)

Team up!
15 min

$$^1H + {}^1H \longrightarrow {}^2H + e^+ + \nu$$

Energy release: 0-0.42 MeV

- The cross-section for scattering neutrinos with energies ~MeV is $\sigma = 10^{-44}\,cm^2$
- A rough density of particles in the Sun is $n \sim 10^{26}\,cm^{-3}$

$$l_{\text{mean free path}} = \frac{1}{\sigma n}$$

1. What happens with the neutrinos in the Sun?
2. What happens with the balance of equation 3 of stellar structure?

Team up!
15 min

phase	T (10^6 K)	total E_{gr}/n	main reactions	total E_{nuc}/n	M_{min}	γ (%)	ν (%)
grav.	$0 \rightarrow 10$	~ 1 keV/n				100	
nucl.	$10 \rightarrow 30$		$^1H \rightarrow {}^4He$	6.7 MeV/n	$0.08\ M_\odot$	~95	~5
grav.	$30 \rightarrow 100$	~ 10 keV/n				100	
nucl.	$100 \rightarrow 300$		$^4He \rightarrow {}^{12}C, {}^{16}O$	≈ 7.4 MeV/n	$0.3\ M_\odot$	~100	~0
grav.	$300 \rightarrow 700$	~ 100 keV/n				~50	~50
nucl.	$700 \rightarrow 1000$		$^{12}C \rightarrow$ Mg, Ne	≈ 7.7 MeV/n	$1.1\ M_\odot$	~0	~100
grav.	$1000 \rightarrow 1500$	~ 150 keV/n					~100
nucl.	$1500 \rightarrow 2000$		$^{16}O \rightarrow$ S, Si	≈ 8.0 MeV/n	$1.4\ M_\odot$		~100
grav.	$2000 \rightarrow 5000$	~ 400 keV/n	Si $\rightarrow \ldots \rightarrow$ Fe	≈ 8.4 MeV/n			~100

Gravitational Energy/nucleon

Nuclear Energy /nucleon

Fraction of energy emitted as γ or ν

Class 3 Review:

- Equation of state stellar gas $P = f(\rho, T, X_i)$ + Distribution function $f(\underline{x}, \underline{p})$

- Equation of state stellar gas

  - Classical / Non-degenerated / Relativistic/Non-relativ. : $P_{gas} = \frac{R}{\mu}\rho T$ ($\mu \rightarrow$ mixture of gases)

  - Quantum / Degenerated
    - Non Relat. : $P_e = 10^{13}\left(\frac{\rho}{\mu_e}\right)^{5/3}\left(\frac{erg}{cm^3}\right)$
    - Relat. : $P_e = 1.2 \times 10^{15}\left(\frac{\rho}{\mu_e}\right)^{4/3}\left(\frac{erg}{cm^3}\right)$

Temperature independent

Class 3 Review:

- Nuclear reaction chains:

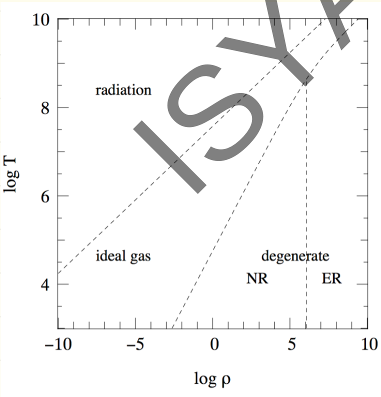
① H → He — via pp cycle ($T < 1.4 \times 10^7$ K)

— via CNO cycle ($T > 1.4 \times 10^7$ K)

② He → ^{12}C, ^{16}O, ^{20}Ne ($T > 10^8$ K)

③ Final cycles ($T > 10^9$ K)

- CNO is a catalyst cycle!
- pp-branches use available ingredients → relation with T



log ρ, log T plane

Equilibrium $\underbrace{\text{structure}}_{}$ of a star of a given composition

solution of a set of differential equations } Stellar Structure Equations

Non-linear equations
Time-dependent → Numerical procedure → Stellar Evolution codes

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}$$

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^2} - \frac{1}{4\pi r^2}\frac{d^2 r}{dt^2}$$

$$\frac{dT}{dm} = -\frac{GmT}{4\pi r^4 P}\nabla \qquad \nabla \begin{cases} \text{radiation} \\ \text{convection} \\ \text{conduction} \end{cases}$$

$$\frac{dl}{dm} = \epsilon_{nuc} - \epsilon_{\nu} - \epsilon_{grav}$$

$\frac{dX_i}{dt}$ = composition change at a given mass shell (if internal mixing consider redistribution)

Solution on interval $0 \leq m \leq M$ for $t > t_0$
(If mass loss $M(t)$)

Boundary conditions $\begin{pmatrix} m=0 \\ m=M \end{pmatrix}$ + Initial Conditions (e.g., $X_i(m, t_0)$)

Evolution description → Conceptual

shorcut 1:

If complete equilibrium (hidrostatic / thermal)

⇓

Ordinary differential equations independent t

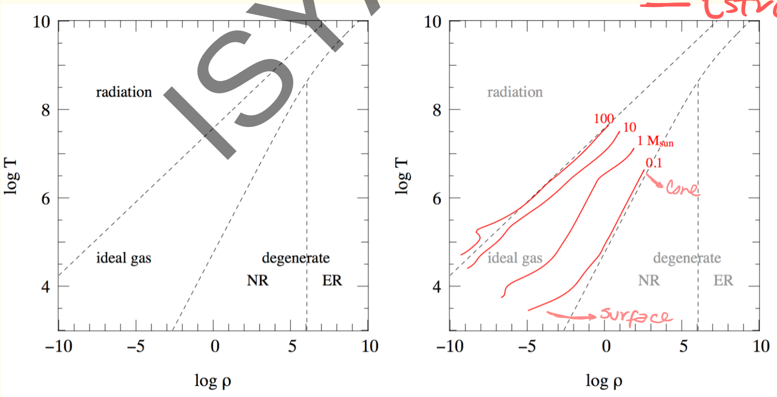
⇓

Initial composition profiles $X_i(m, t_0)$ → initial conditions

⇓

Zero Age Main Sequence
ZAMS

zero-age (homogeneous)
— Estructure models



shorcut 2:

Polytropes:

The uniform assumption

$$P(r) = K\rho(r)^{\Gamma}$$

A. Non-relativistic degenerate gas (white dwarfs, brown dwarfs)

$$P = 10^{13}\left(\frac{\rho}{\mu_e}\right)^{5/3}\left(\frac{dyne}{cm^2}\right)$$

B. Relativistic degenerate gas (extreme white dwarfs)

$$P = 1.2\times10^{15}\left(\frac{\rho}{\mu_e}\right)^{4/3}\left(\frac{dyne}{cm^2}\right)$$

C. Convective stars

Movement equation for polytropes

Hydrostatic equilibrium + mass continuity + polytrope P vs. ρ relation + $\tilde{\rho} = \rho/\rho_0$, $\tilde{r} = \frac{r}{r_0}$

$$r_0^2 = \frac{k\Gamma}{4\pi G}\,\rho_0^{\Gamma-2}$$

$$\frac{d}{d\tilde{r}}\left(\tilde{r}^2 \tilde{\rho}^{\,\Gamma-2} \frac{d\tilde{\rho}}{d\tilde{r}}\right) = -\tilde{r}^2 \tilde{\rho}$$

Lane-Emden equation

With the proper initial conditions $\longrightarrow$ M, R, ρ relations

$$M = \left[\frac{4\pi \tilde{R}^3 (k\Gamma)^{3/2}}{(4\pi G)^{3/2}(\Gamma-1)} D(\Gamma)\right] \rho_0^{\frac{3\Gamma-4}{2}} \qquad R = \left[\frac{k\Gamma}{4\pi G}\tilde{R}^2\right] \rho_0^{\Gamma-2}$$

where,

$$D(\Gamma) = \left[-\frac{1}{\tilde{R}_*}\frac{d}{d\tilde{r}}\left(\tilde{\rho}^{\,\Gamma-1}\right)\right]_{\tilde{r}=\tilde{R}_0}$$

If monoatomic gas,

$\Gamma = \frac{5}{3}$, $M \propto \rho_0^{1/2}$, $R \propto \rho_0^{-1/6} \propto M^{-1/3}$

Direct application →

Chandrasekhar:

Relativistic $\Gamma = 4/3$ — Non-relativistic $\Gamma = 5/3$

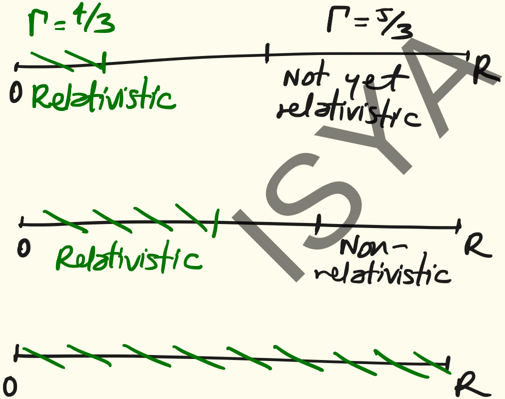
$$P \neq K\rho^{\Gamma}$$

Unique M_{Lim} value with hydrostatic equilibrium in a relativistic degenerate gas

$t=0$

$t>0$

t_{final}



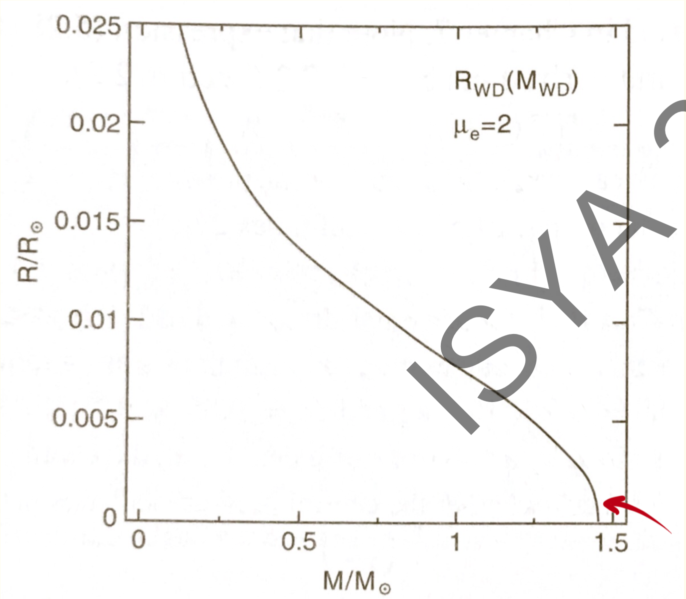
$$M_{Lim} = M_{Chandrasekhar} = \frac{5.83}{\mu_e^2} M_{\odot}$$

If $M > M_{Lim}$ there is no longer equilibrium and the star collapses.

Real life value, star $M < 8 M_\odot$

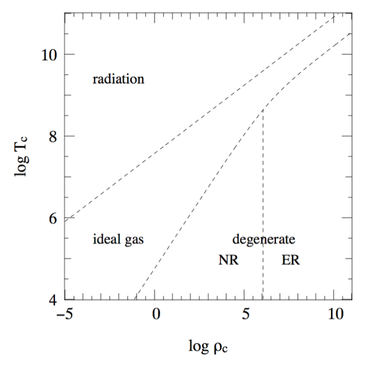
A star with He, C, O ionized $\longrightarrow$ $\mu_e \approx 2$ (white dwarf)

$M_{chandrasekhar} = 1.46 M_\odot$



There are not white dwarfs with $M > M_{ch}$.

CO cores unable to ignite



Center

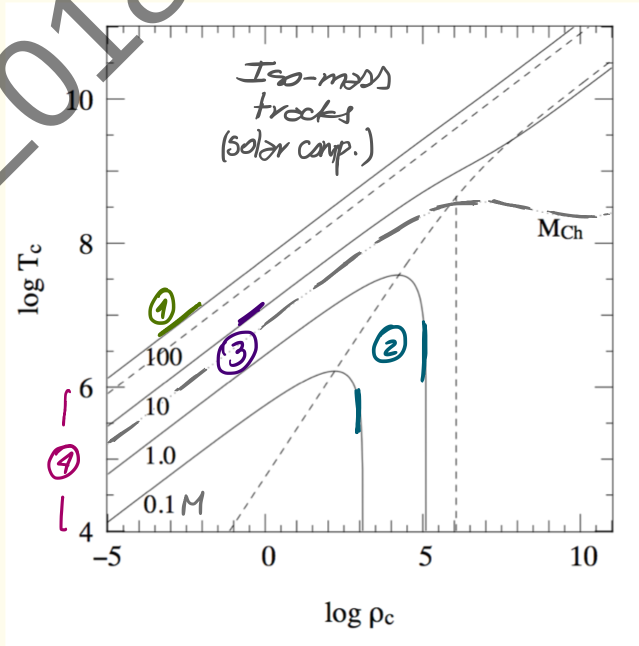
Most evolved part of a star
Set pace of evolution

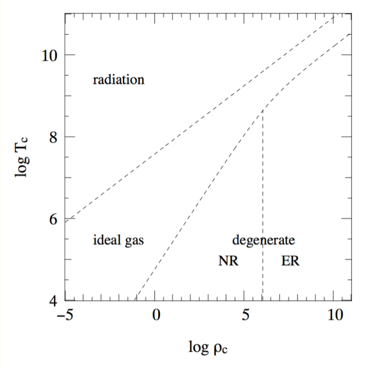
① Larger mass → more relevant radiation pressure.
→ radiation pressure relevance stays as star contracts.

② $M < M_{Ch}$ → relevant degenerate non-relativistic e^-.
→ T independent

③ $M > M_{Ch}$ → Pressure dominated by ideal gas.

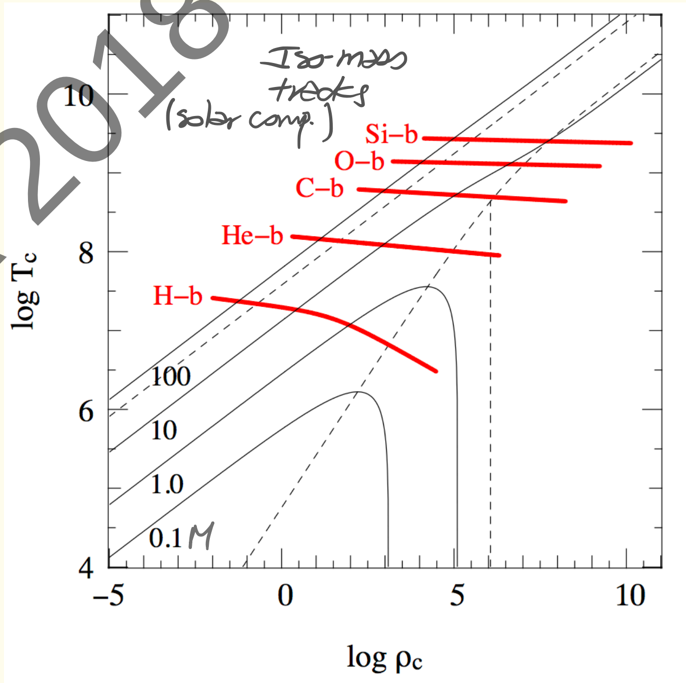
④ Mass range ~0.073–100 $M_\odot$

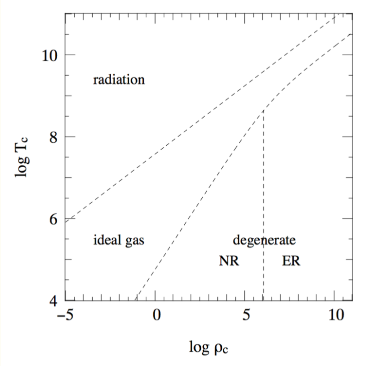




Center

Most evolved part of a star
Set pace of evolution





Center

Most evolved part of a star

Set pace of evolution

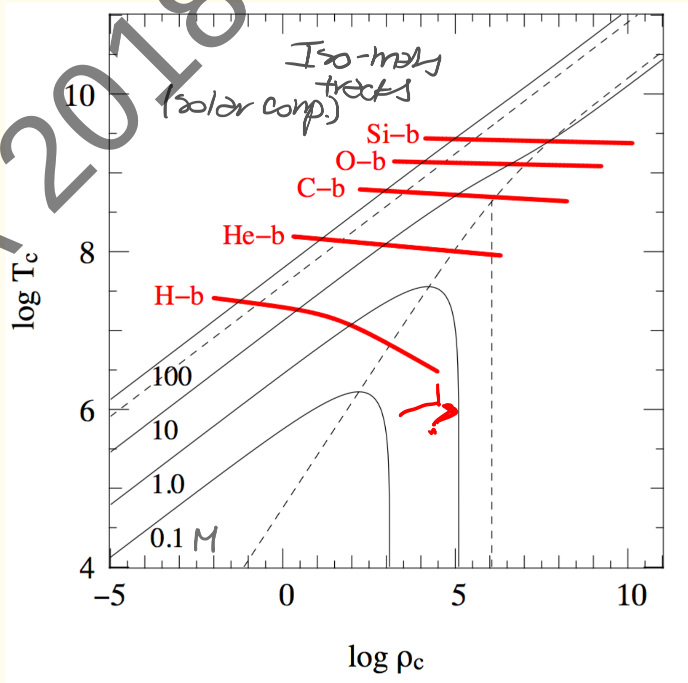
Partner discussion:

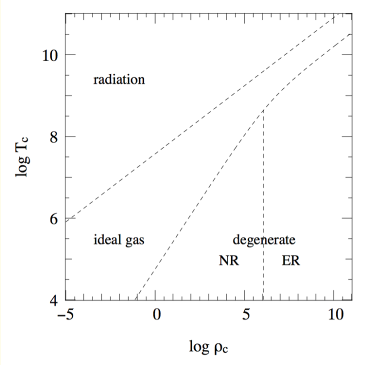
What is going on???

Hint: $X = 0.70$

$Y = 0.28$

$Z = 0.02$





Center

Most evolved part of a star

Set pace of evolution

Partner discussion:

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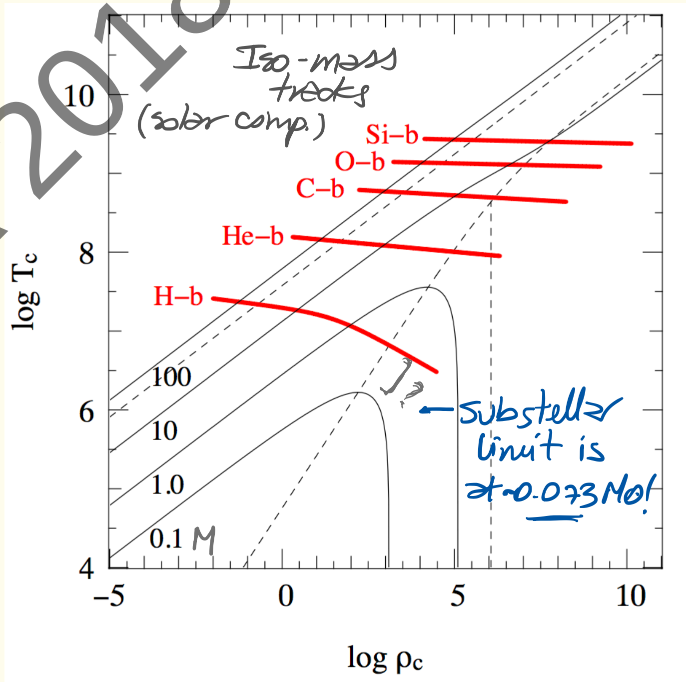
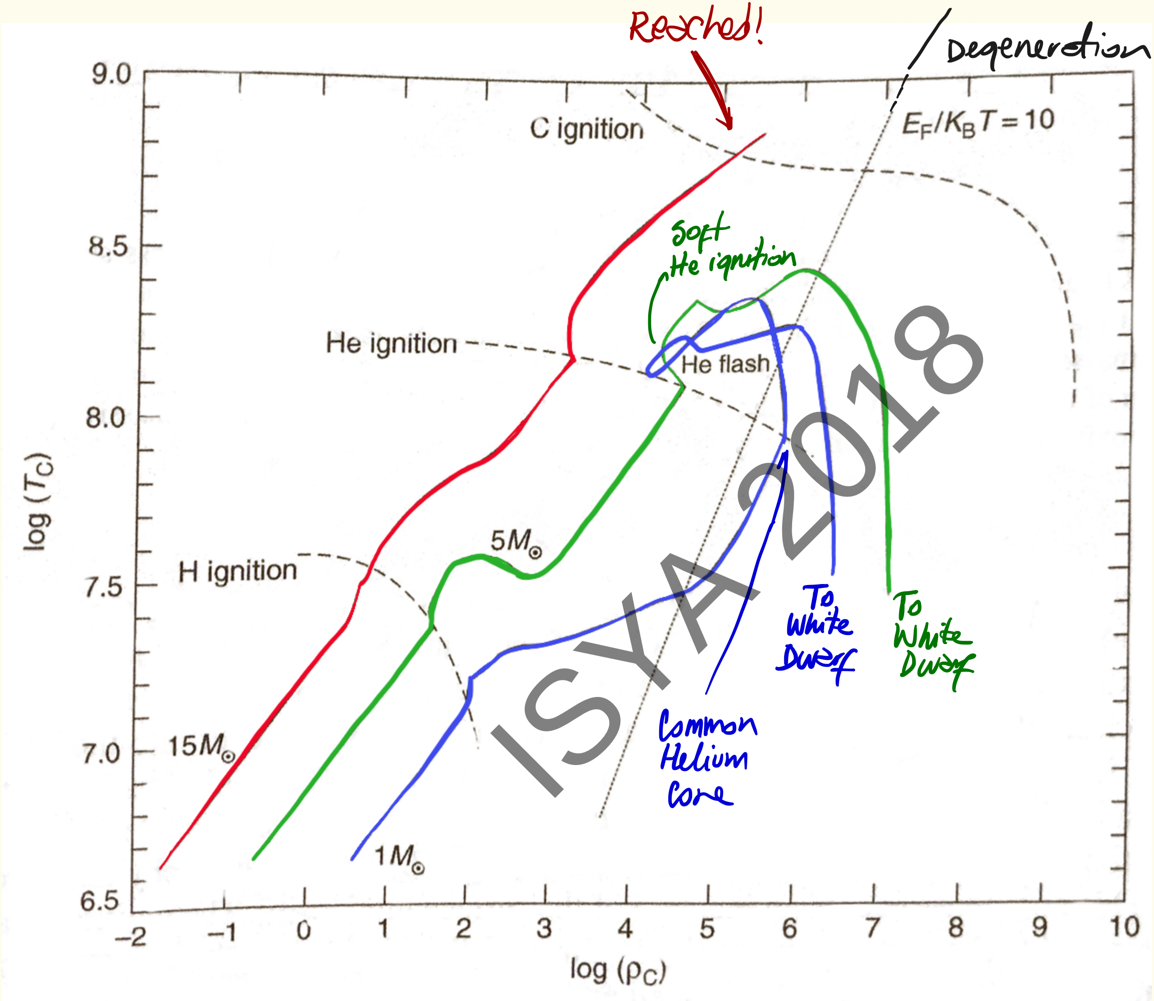


Minimum mass for He-ignition

$\approx 0.3\,M_\odot$

$$\tau_{He\text{-}b} \approx 0.1\,\tau_{H\text{-}b}$$



$M_{core} = 0.3 M_\odot$

↓

He flash at the external shells

There is no predictive theory for star formation

(ρ, T) molecular cloud $\xrightarrow{?}$ Star Formation Efficiency $\xrightarrow{?}$ Initial Mass Function

Knowledge gap but...

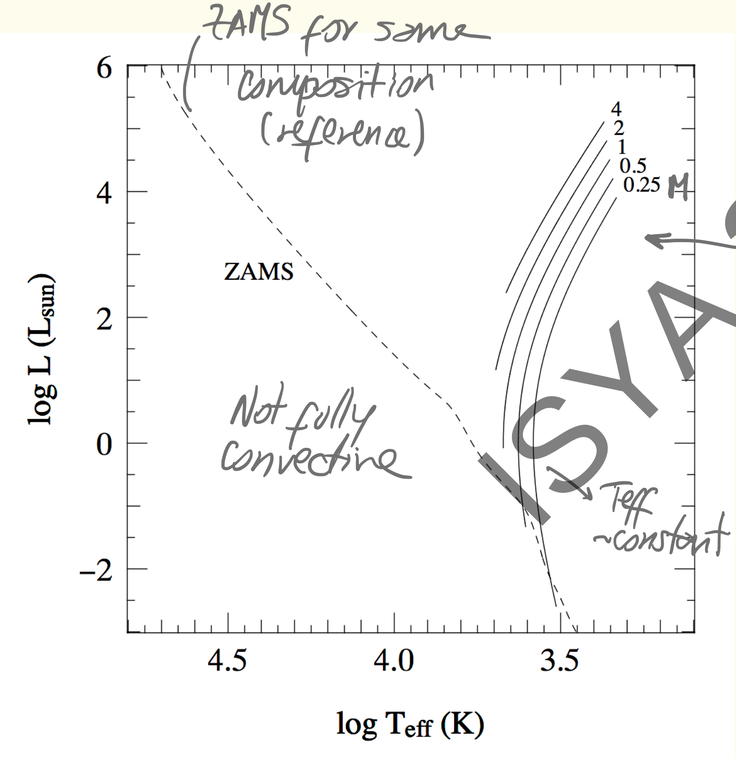
Once star is in complete equilibrium $\longrightarrow$ Main Sequence $\longrightarrow$ 4 structure eqst. Initial composition

Formation details are wiped out by evolution

$\hookrightarrow$ let's <u>pause</u> this!

Early stages:

HR Diagram → Hayashi lines



Proto-star : $T \sim 8\times10^4$ K

↓

High opacities

↓

Radiative energy transport inefficient

↓

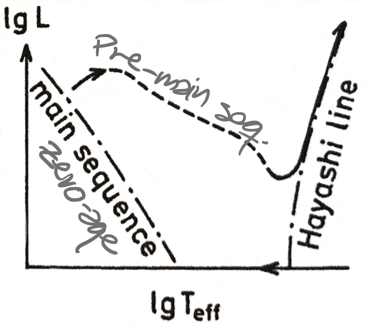
Convective proto-star

Early stages:

HR Diagram → Pre-main sequence

Not yet H-burning

luminosity → Gravitational contraction



$M/M_\odot = 0.5 \rightarrow \Delta t_{1-2} \simeq 10^8$ yr

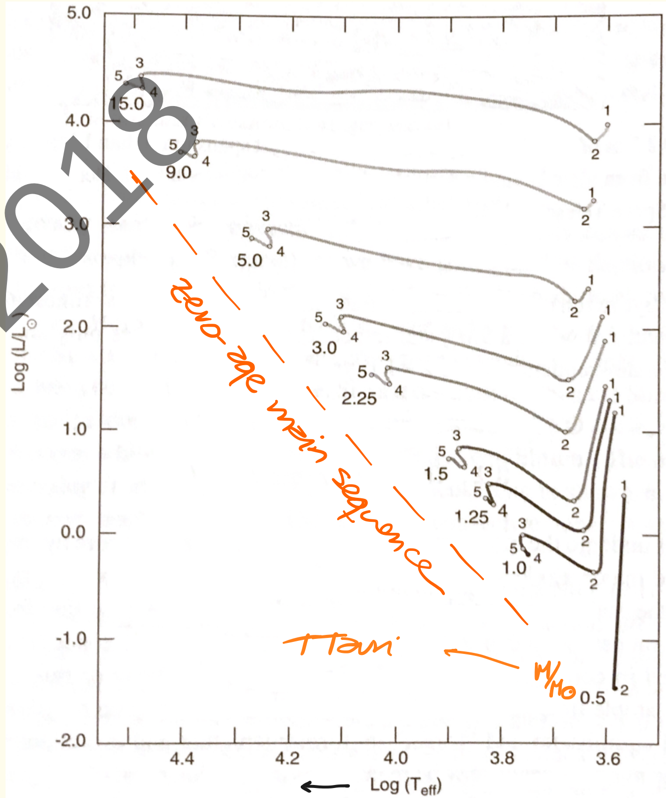
$M/M_\odot = 15 \rightarrow \Delta t_{1-2} \simeq 10^2$ yr

$2\text{-}3 \simeq 10^4$ yr

$3\text{-}4 \simeq 10^4$ yr

$4\text{-}5 \simeq 10^3$ yr

$1 M_\odot$ spends 3×10^7 yr PMS and 10^{10} yr in MS



Early stages:

HR Diagram → Pre-main sequence

Not yet H-burning

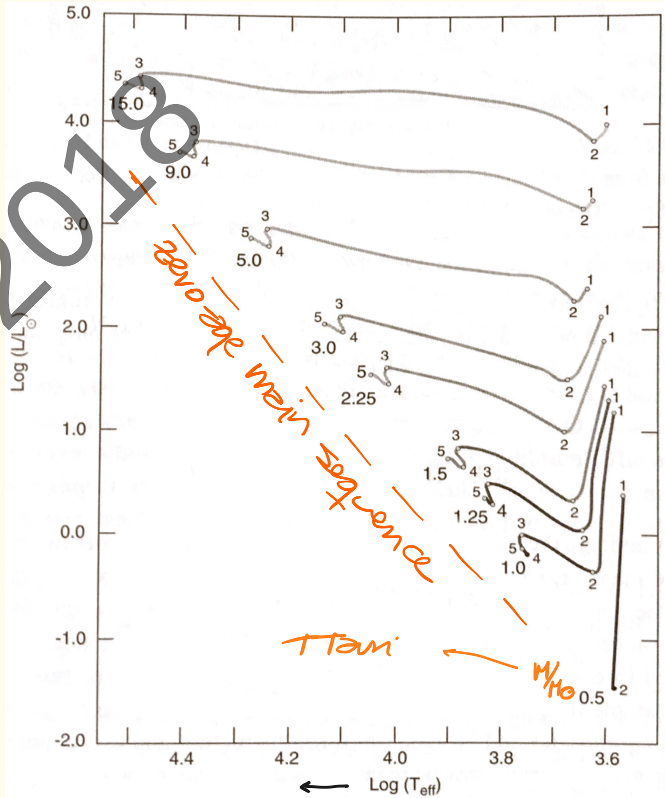
Luminosity → Gravitational contraction

1-2: Fully convective, contracts, L ↓

2-3: Since T ↑, opacity ↓, a radiative core develops.
Leave Hayashi line.
Convective envelope ↓
Radiative core ↑

3-5: $^1H + {}^1H \longrightarrow {}^2H + e^+ + \nu$ $T \sim 1 \times 10^6 K$
$^{12}C \longrightarrow {}^{14}N$
Wiggles

5: T_c high enough H-burning - ZAMS
Only if $M > 0.073 M_\odot$

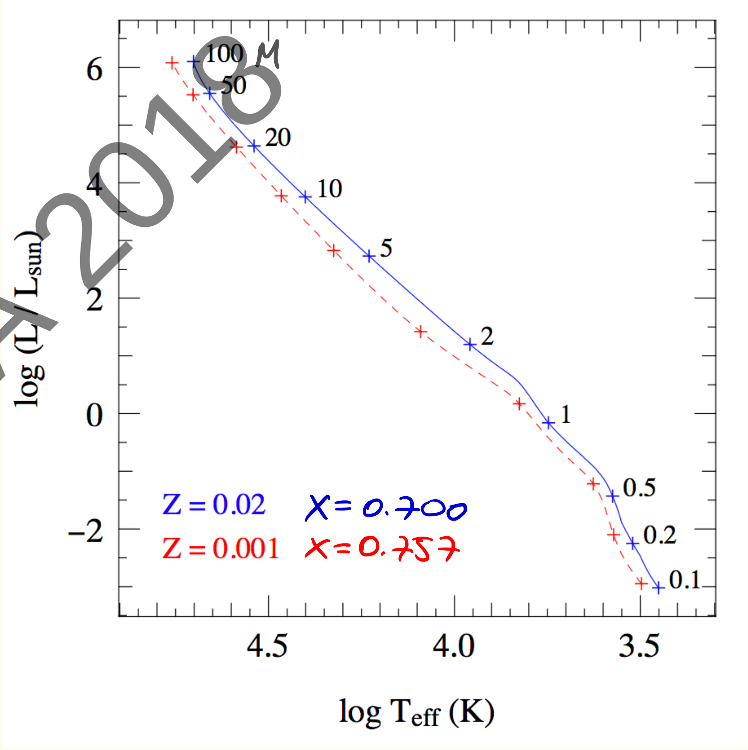


Early stages:

HR Diagram → zero age main sequence

ZAMS models for metal-poor stars are bluer - - - -

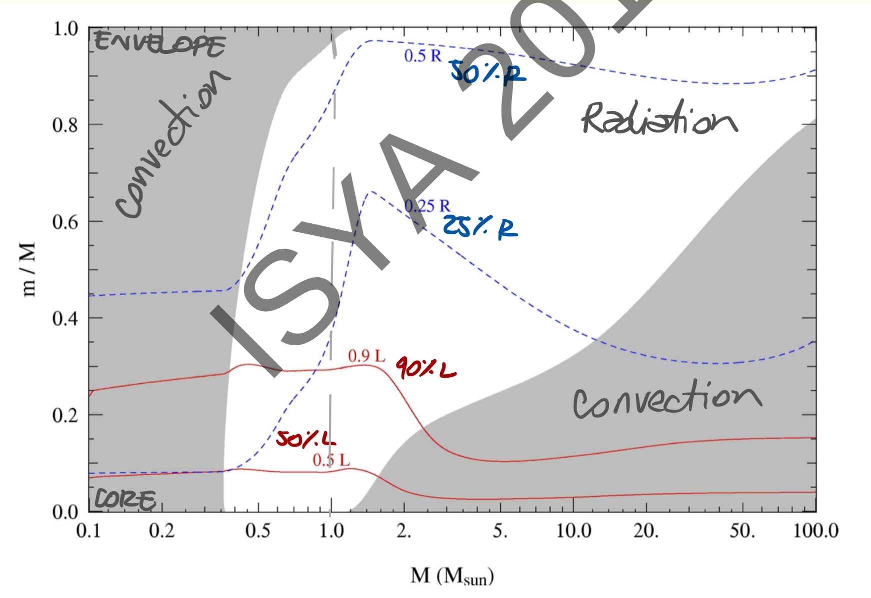
Metal-poor low mass stars are more luminous than their metal rich counterparts +



Early stages:

Convective regions in ZAMS

- Completely convective $M < 0.35 M_\odot$
- Radiative core + convective envelope $0.35 M_\odot < M < 1.2 M_\odot$
- Convective core + radiative envelope $M > 1.2 M_\odot$



Hydrogen burning phase:

stars evolve from ZAMS towards higher L and larger R

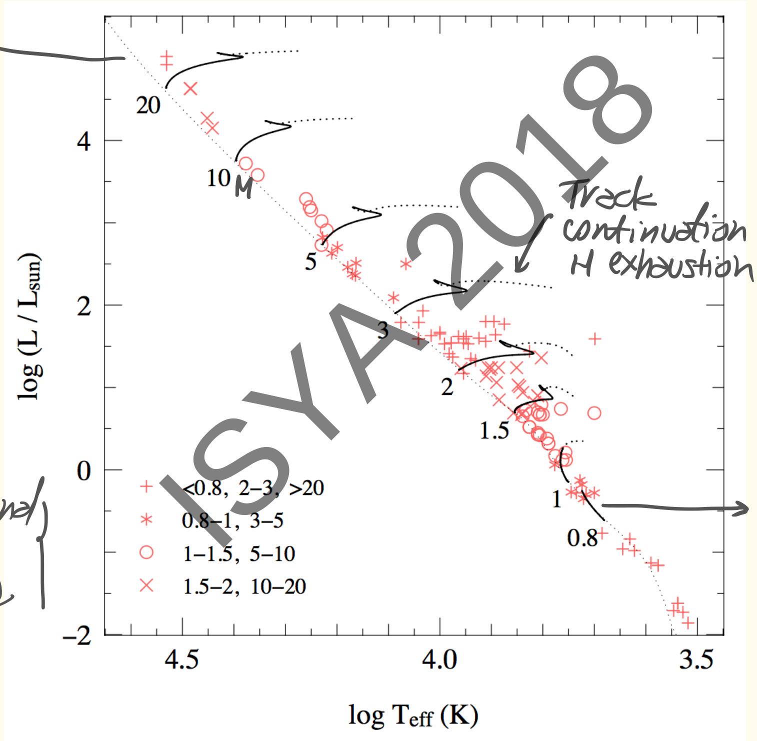
→ Evolved stars

ZAMS

(In low mass star the R increases modestly)

$\tau_{MS} \propto M^{1-3.8}$

$\tau_{MS} \downarrow$ if $M \uparrow$



M

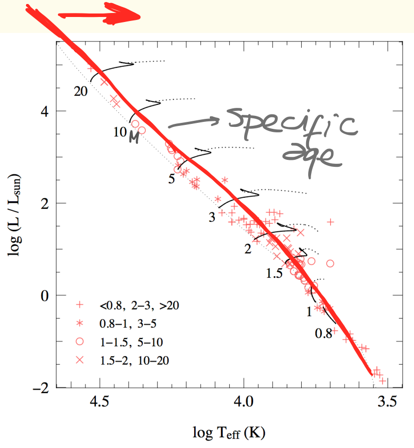
Track continuation H exhaustion

Observational data

Accurate mass, L, R.

$t = \underline{1.4}$ Gyr!

Hydrogen burning phase:



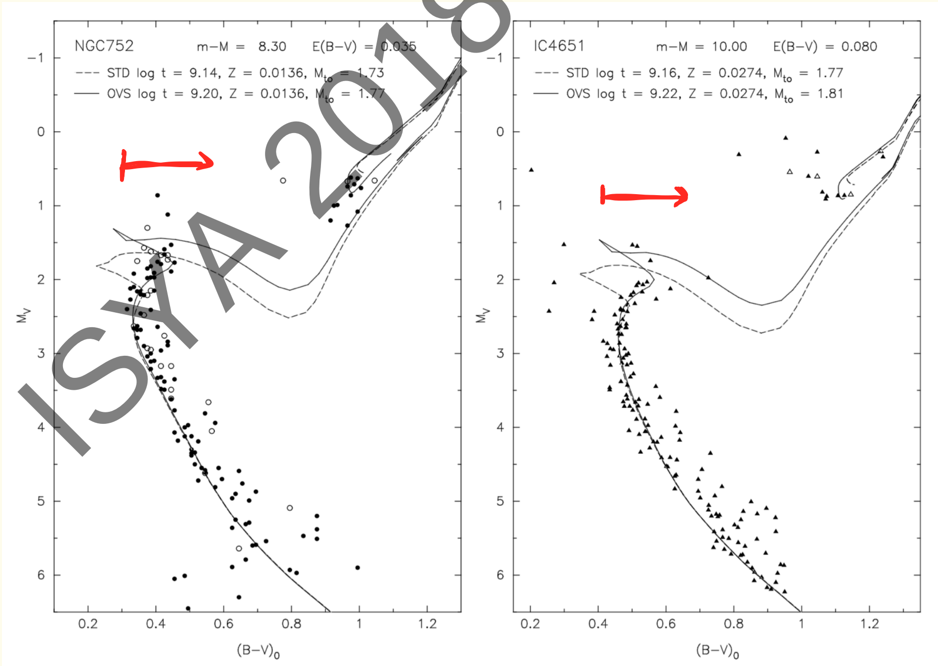
→ specific age

Turn-off point

Cluster dating

§ coeval sources τ_{cl}

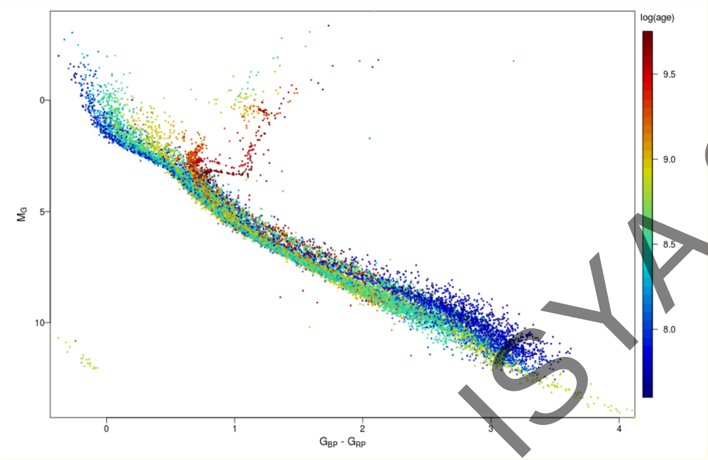
Above certain mass $\tau_{ms} < \tau_{cl}$ left main sequence



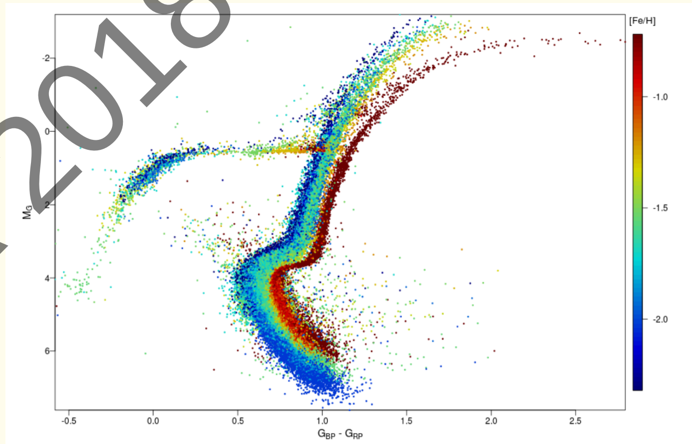
Gaia Data Release 2:
Observational Hertzsprung-Russell diagrams

Gaia Collaboration, C. Babusiaux[1,2], F. van Leeuwen[3], M.A. Barstow[4], C. Jordi[5], A. Vallenari[6], D. Bossini[6], A. Bressan[7], T. Cantat-Gaudin[6,5], M. van Leeuwen[3], A.G.A. Brown[8], T. Prusti[9], J.H.J. de Bruijne[9], C.A.L. Bailer-Jones[10], M. Biermann[11], D.W. Evans[3], L. Eyer[12], F. Jansen[13], S.A. Klioner[14], U. Lammers[15], L. Lindegren[16], X. Luri[5], F. Mignard[17], C. Panem[18], D. Pourbaix[19,20], S. Randich[21], P. Sartoretti[2], H.I. Siddiqui[22], C. Soubiran[23], N.A. Walton[3], F. Arenou[2], U. Bastian[11], M. Cropper[24], R. Drimmel[25], D. Katz[2], M.G. Lattanzi[25], J. Bakker[15], C. Cacciari[26], J. Castañeda[5], L. Chaoul[18], N. Cheek[27], F. De Angeli[3], C. Fabricius[5], R. Guerra[15], B.

3 months ago!



32 open clusters!

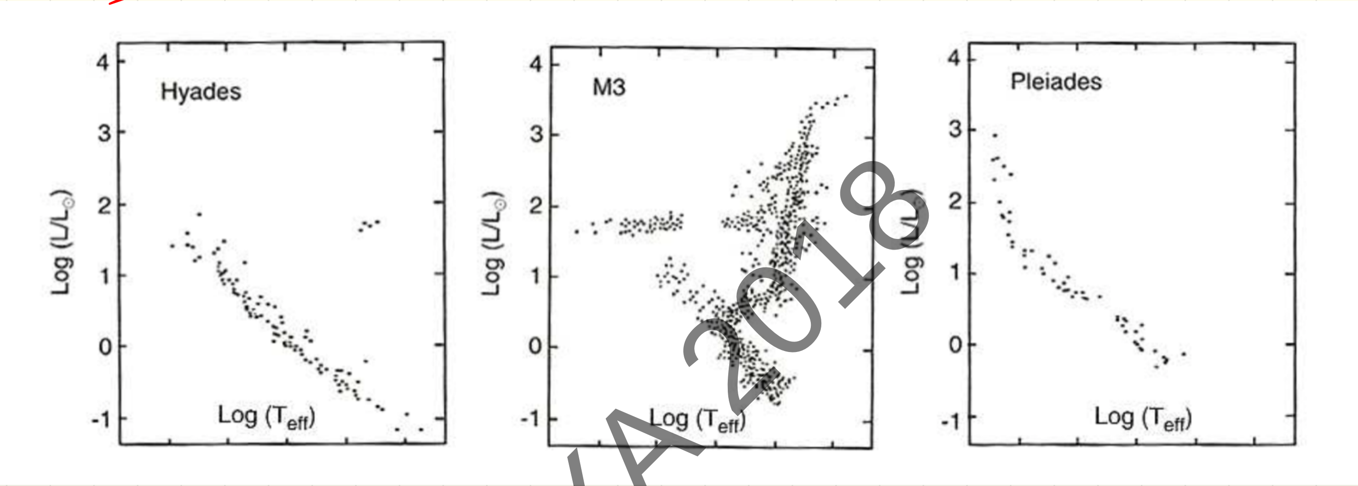


14 globular clusters!

Check dispersion!

Wake up!

Which cluster is younger/older?



t

A. Hyades, Pleiades, M3

B. M3, Hyades, Pleiades

C. Pleiades, M3, Hyades

D. Pleiades, Hyades, M3

E. They are all about the same age

Post-main sequence:

Core → Hydrogen exhausted
Envelope → Hydrogen rich

At this stage:

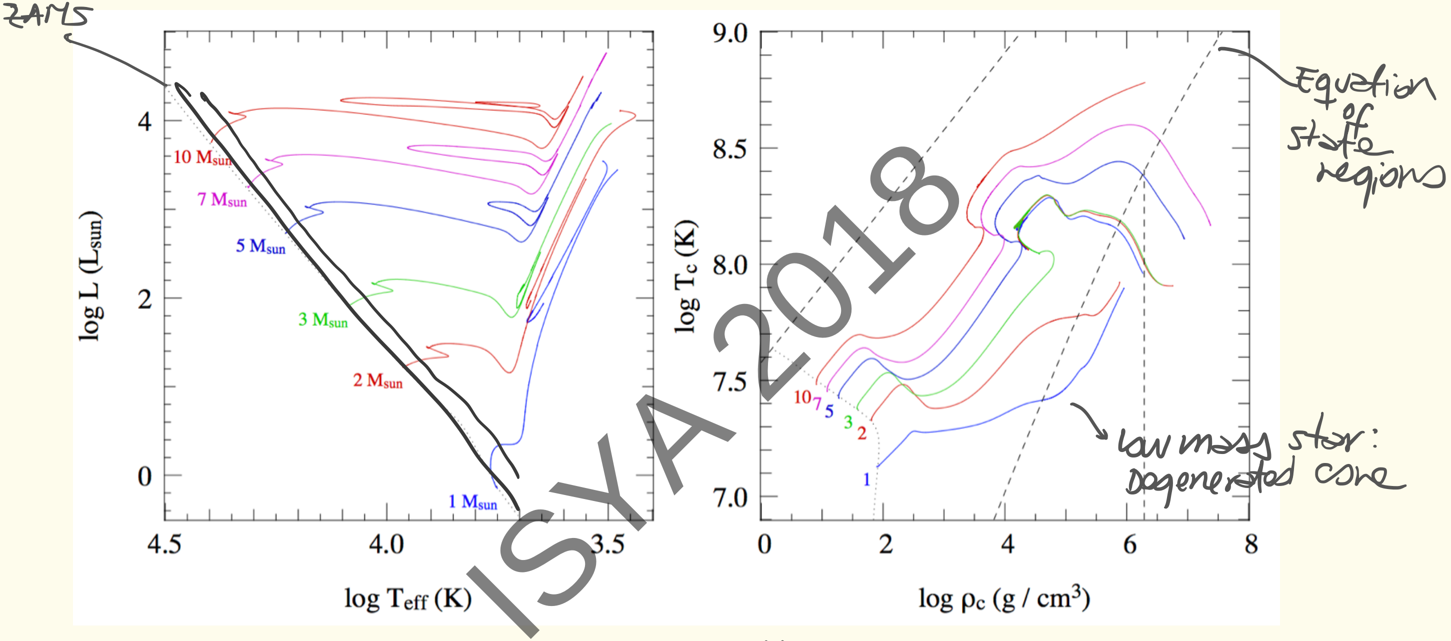
- Low mass stars: ~0.8-2 $M_\odot$ — Degenerate helium core → red giant branch. He ignition is unstable → He flash.

- Intermediate mass stars ~2-8 $M_\odot$: Non-degenerated helium core. Stable He ignition. When He is exhausted → CO core degenerated.

- Massive stars: ~>8 $M_\odot$ — C ignition non-degenerated core. If $M \gtrsim 11 M_\odot$ ignition heavier elements until Fe core

Post-main sequence:

core + envelopes



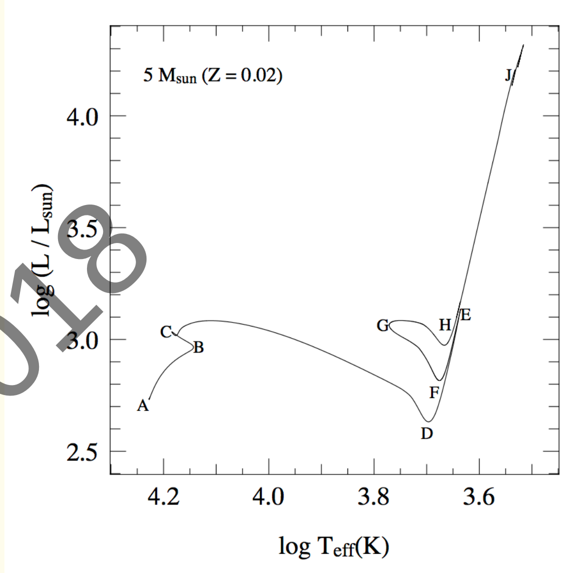
If star has an active shell-burning source, the shell acts as a mirror between core and envelope

core contraction → envelope expansion

core expansion → envelope contraction

Post-main sequence:

Star $5M_\odot$



AB – 2×10^7 y

$A \longrightarrow$ ZAMS

Convective core

$T_c, \rho_c \uparrow$ with ϵ_{CNO}

BC – 0.5×10^7 y

Structure of nuclear fusion in shells. Core He burning

CD – 0.4×10^7 y

Leaves the MS

Outer shells expanded

$T_c \uparrow > 10^8$ K, He $\longrightarrow$ C

Go to red giant zone

DE – 0.1×10^7 y

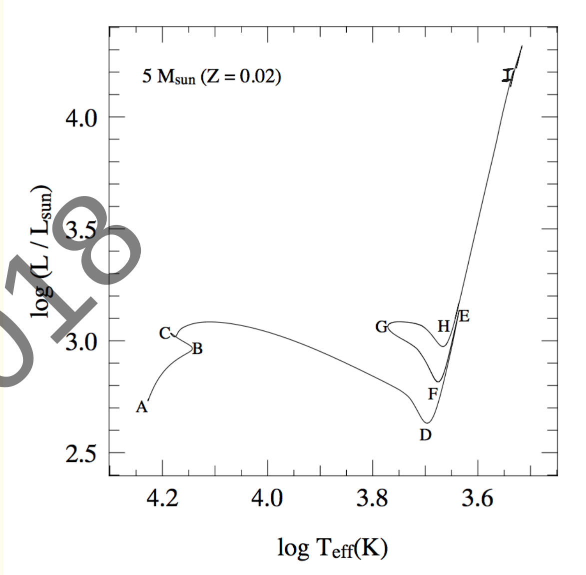
Vertical trajectory

Convective envelope

$T_{eff} \downarrow$

Post-main sequence:

Star 5 $M_\odot$



EF 0.4×10^7 y
$T_{eff}\uparrow$, convective shell disappear
External envelope deflates

FG 0.2×10^7 y
Double He-burning shell

GH Core of degenerate gas
He-burning shells contract

HI Core contraction
Internal shells expand
$T\downarrow$, convection layers disappear
Go to the red giand branch

Post-main sequence:

Star $5M_\odot$

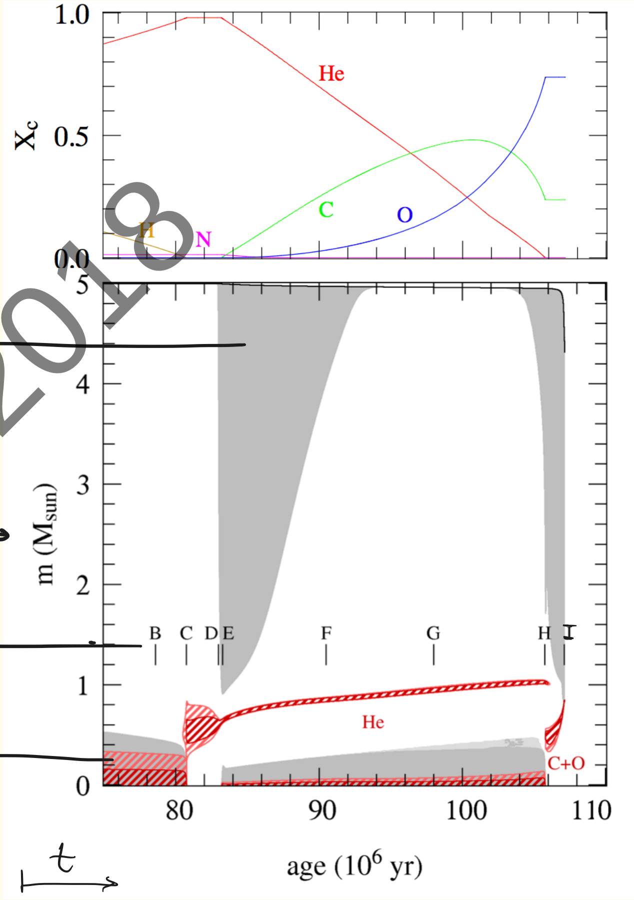
Central Mass Fractions of elements

Convective regions

Mass coordinate

Evolutionary stages

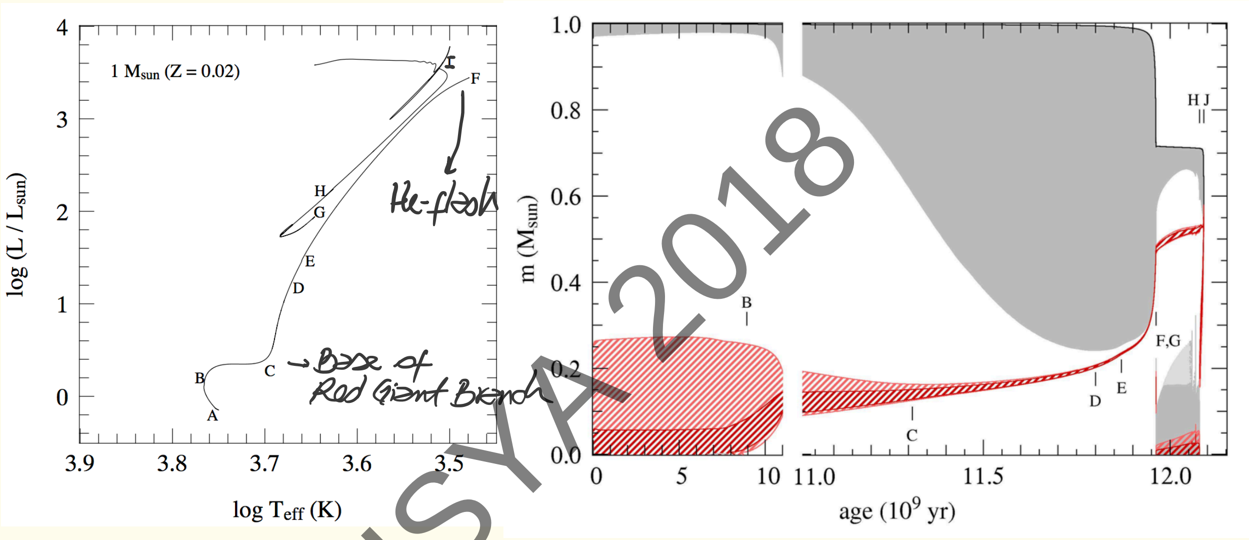
Nuclear energy generation



Post-main sequence:

Star 1 $M_\odot$

Sun!



AB 9×10^9 yr
H exhausted

BC Core contraction
Envelope expansion
Subgiant branch

C Degenerated He core
Largely convective envelope
He core grows

GH He-burning
~ 120 Myr

He-burning: $T > 10^8$ K

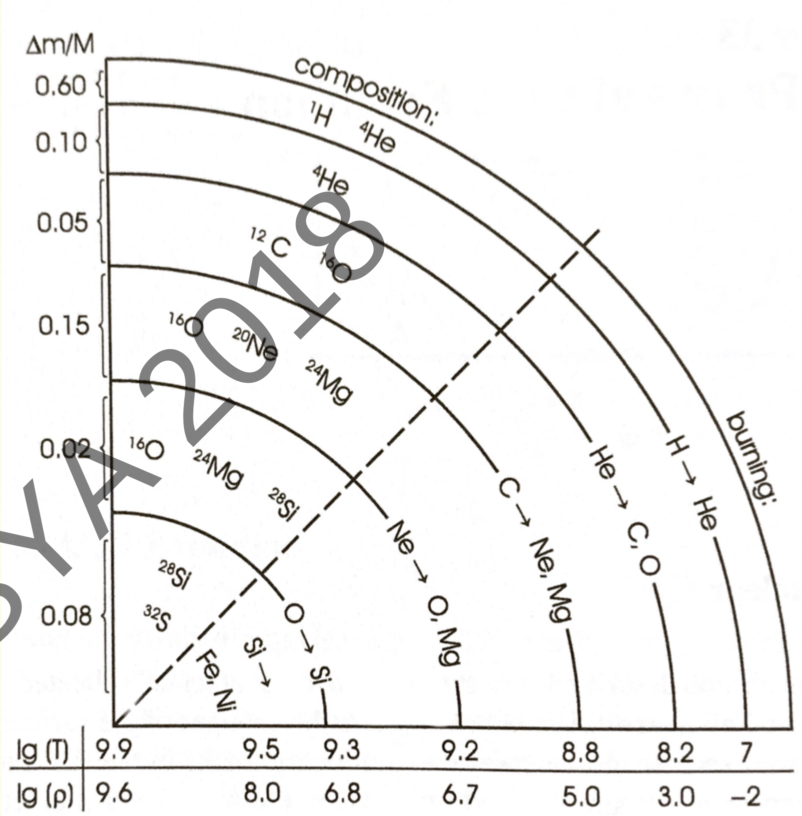
- Very low mass stars never begin He-burning.

Helium flash:
- Explosive ignition of He
- Very dynamic stage
- Core expands rapidly after He flash
- Envelope contracts
- T ↓, L ↓ (expansion)

→ Horizontal branch → "He main sequence"

Again: He core exhaustion, then He-shell burning
H-shell still there → double shell burning
Go to the asymptotic giant branch
C core forms...

"Onion" structure of a highly evolved star

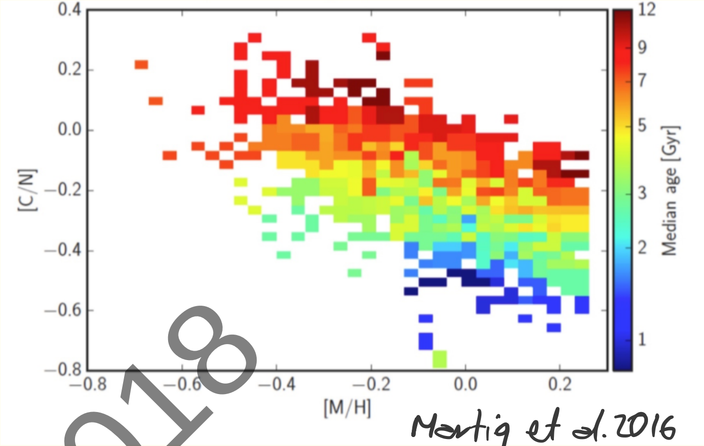


A suddenly deepned convention zone can "dredge-up" heavier elements from lower layers → complex convection

"Dredge-up" modify [C/N] in the observed spectra

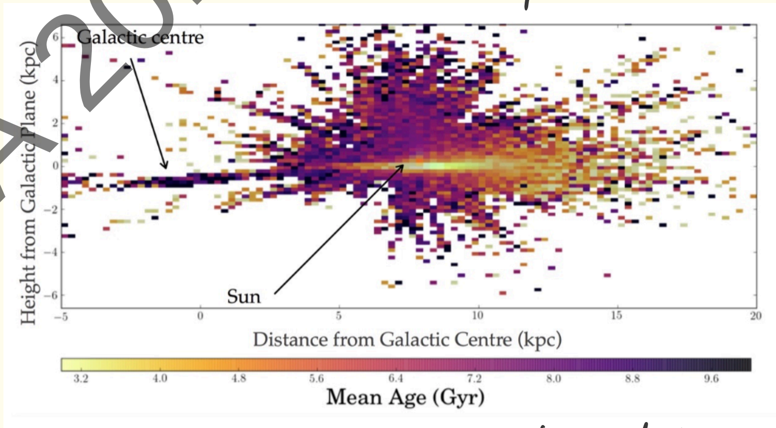
Direct application:

Measure ages of red giant stars from spectroscopy

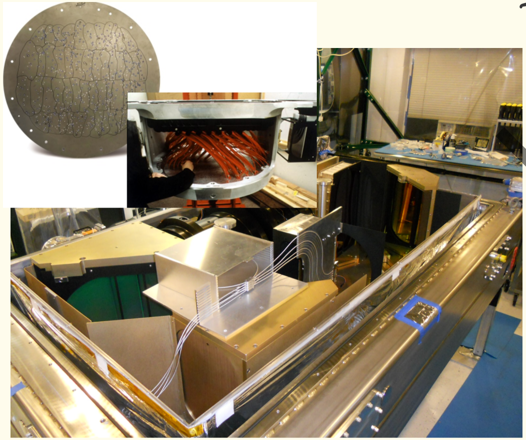


Martig et al. 2016

Age map of the Milky Way



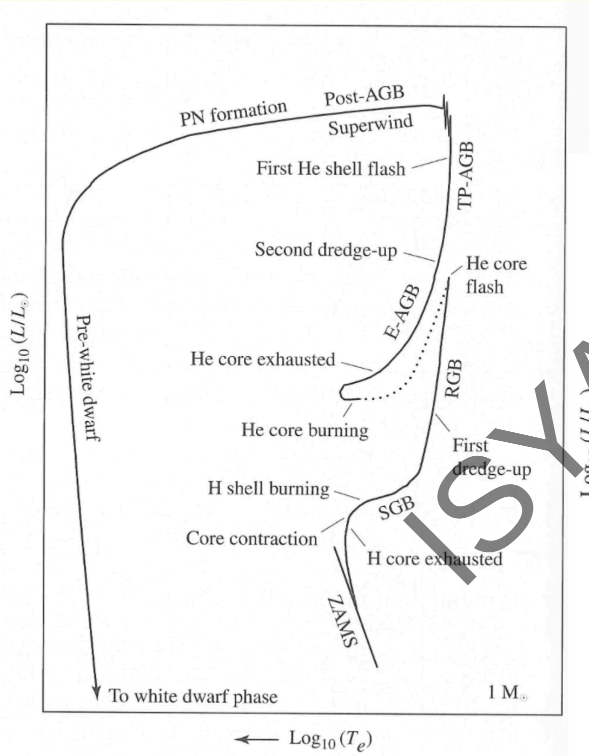
Ness et al. 2016



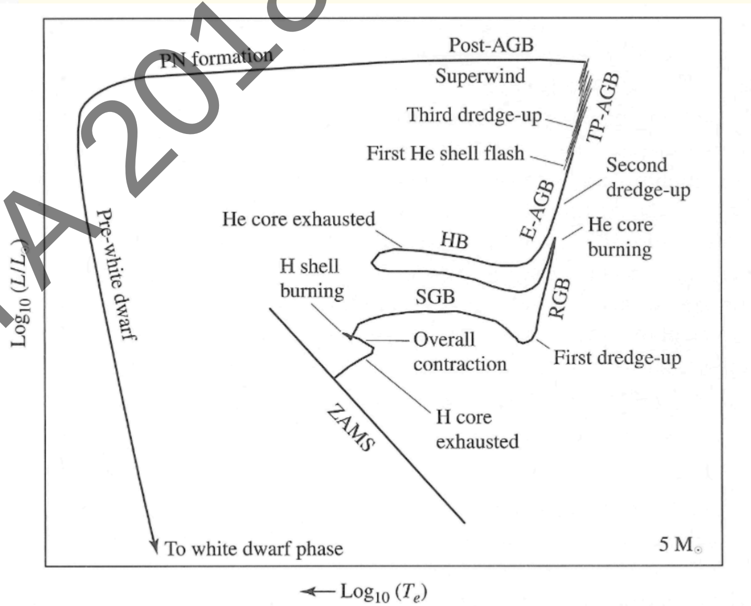
APOGEE data

Schematic steps

SGB : subgiant branch
RGB : red giant branch
E-AGB : early asymptotic giant branch
TP-AGB : thermal pulse AGB
Post-AGB
PN : planetary nebula
HB : horizontal branch



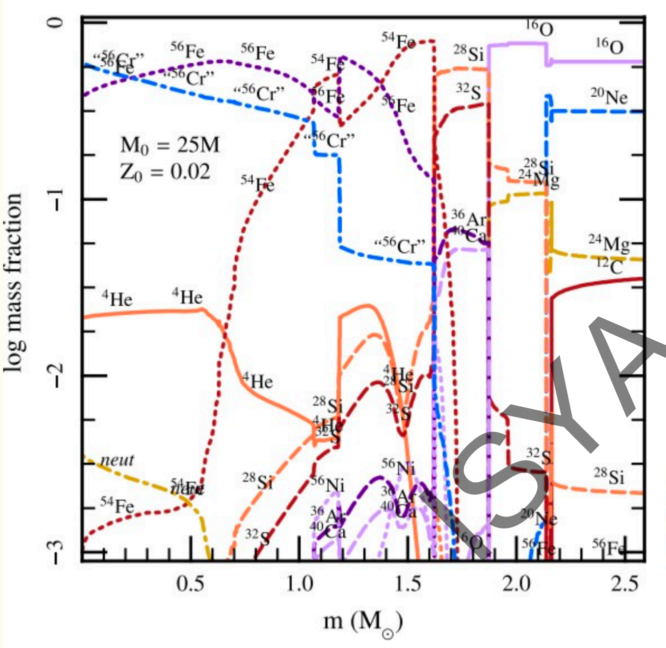
$1 M_\odot$

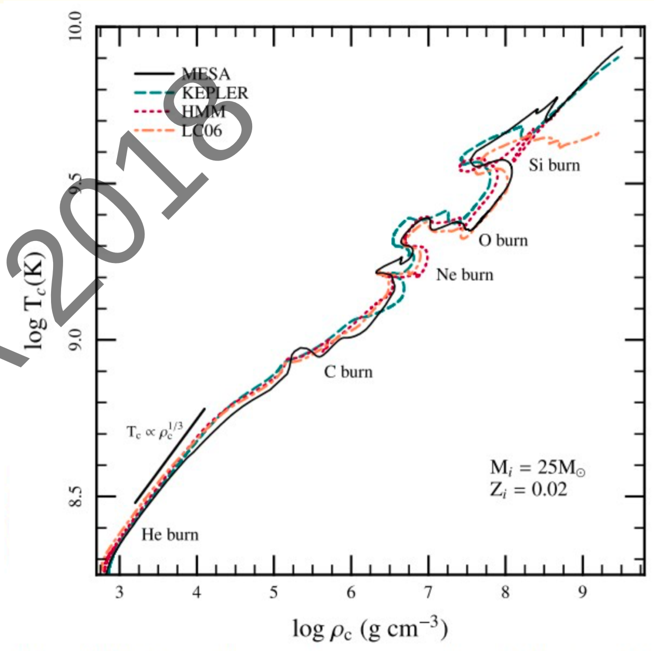


$5 M_\odot$

High mass stars:

25 $M_\odot$





burning stage	T (10^9 K)	ρ (g/cm^3)	fuel	main products	timescale
hydrogen	0.035	5.8	H	He	1.1×10^7 yr
helium	0.18	1.4×10^3	He	C, O	2.0×10^6 yr
carbon	0.83	2.4×10^5	C	O, Ne	2.0×10^3 yr
neon	1.6	7.2×10^6	Ne	O, Mg	0.7 yr
oxygen	1.9	6.7×10^6	O, Mg	Si, S	2.6 yr
silicon	3.3	4.3×10^7	Si, S	Fe, Ni	18 d

phase	T (10^6 K)	total E_{gr}/n	main reactions	total E_{nuc}/n	M_{min}	γ (%)	ν (%)
grav.	$0 \to 10$	~ 1 keV/n				100	
nucl.	$10 \to 30$		$^1H \to {}^4He$	6.7 MeV/n	$0.08\ M_\odot$	~95	~5
grav.	$30 \to 100$	~ 10 keV/n				100	
nucl.	$100 \to 300$		$^4He \to {}^{12}C, {}^{16}O$	≈ 7.4 MeV/n	$0.3\ M_\odot$	~100	~0
grav.	$300 \to 700$	~ 100 keV/n				~50	~50
nucl.	$700 \to 1000$		$^{12}C \to$ Mg, Ne	≈ 7.7 MeV/n	$1.1\ M_\odot$	~0	~100
grav.	$1000 \to 1500$	~ 150 keV/n					~100
nucl.	$1500 \to 2000$		$^{16}O \to$ S, Si	≈ 8.0 MeV/n	$1.4\ M_\odot$		~100
grav.	$2000 \to 5000$	~ 400 keV/n	Si $\to \ldots \to$ Fe	≈ 8.4 MeV/n			~100

Gravitational Energy/nucleon

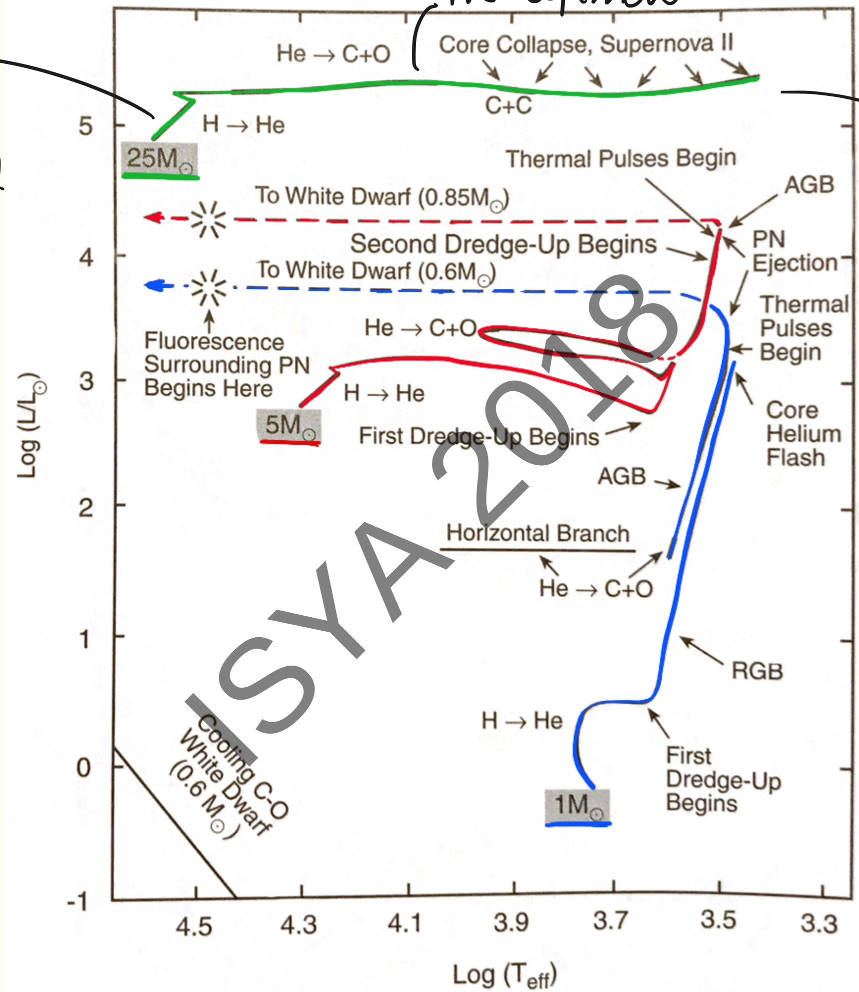
Nuclear Energy /nucleon

Fraction of energy emitted as γ or ν

short main sequence

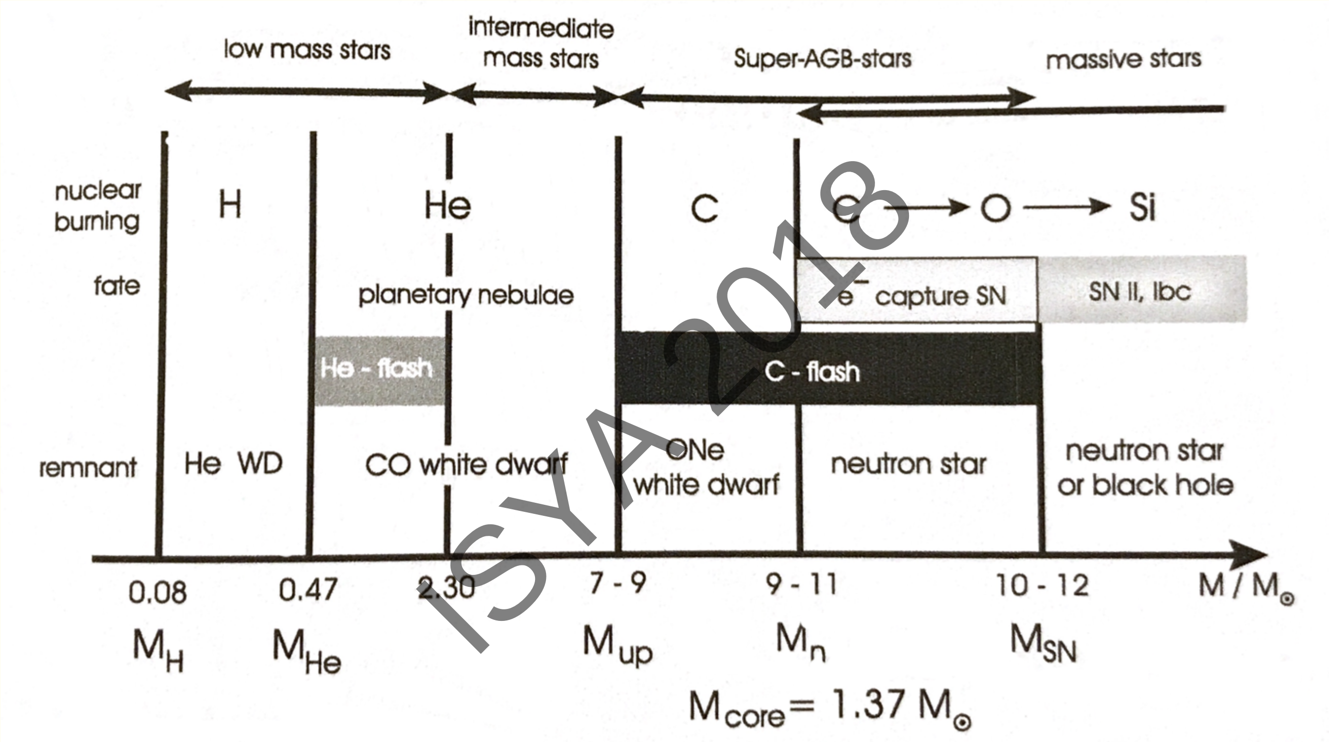
Lw is important for $M > 10 M_\odot$

Pre-supernova



Mass loss due to stellar winds

Fate of different masses:

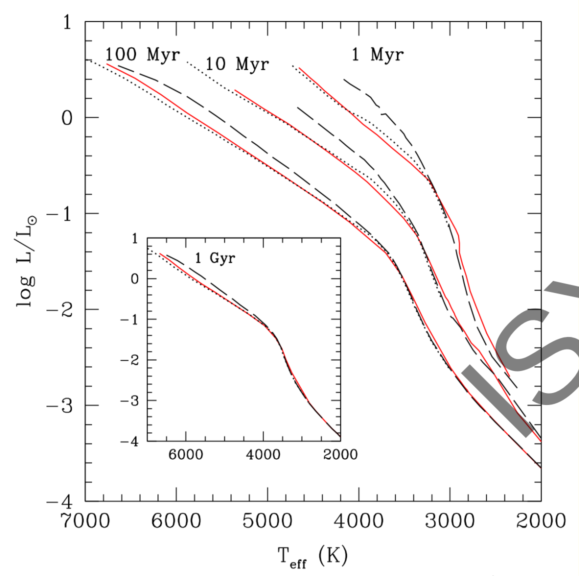


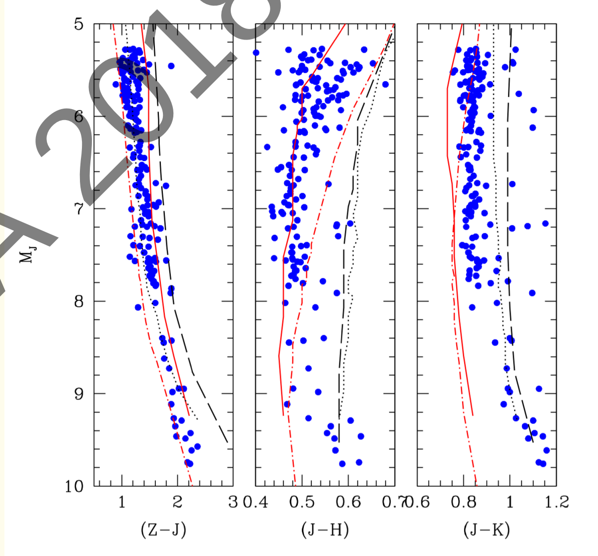
New evolutionary models for pre-main sequence and main sequence low-mass stars down to the hydrogen-burning limit

Isabelle Baraffe[1,2], Derek Homeier[2], France Allard[2], and Gilles Chabrier[2,1]

[1] University of Exeter, Physics and Astronomy, EX4 4QL Exeter, UK (e-mail: i.baraffe@ex.ac.uk)
[2] École Normale Supérieure, Lyon, CRAL (UMR CNRS 5574), Université de Lyon, France (e-mail: derek.homeier@ens-lyon.fr, fallard@ens-lyon.fr, chabrier@ens-lyon.fr)

2015





Comparison between models

Data from Peña Ramirez et al. 2012

Class 4 Review:

- Complete equilibrium (hidrostatic / thermal) + initial composition

$$X_i(M, t_0) \longrightarrow \text{ZAMS}$$

$$P(r) = K\rho(r)^{\Gamma}$$

- T_c, ρ_c plane location $\longrightarrow$ Involved equation of state

Larger mass $\rightarrow$ more relevant radiation pressure
$\rightarrow$ radiation pressure relevance stays as star contracts.

$M < M_{ch} \rightarrow$ relevant degenerate non-relativistic e^-.
$\rightarrow$ T independent

$M > M_{ch} \rightarrow$ Pressure dominated by ideal gas.

Mass range $\sim 0.073 - 100\ M_\odot$

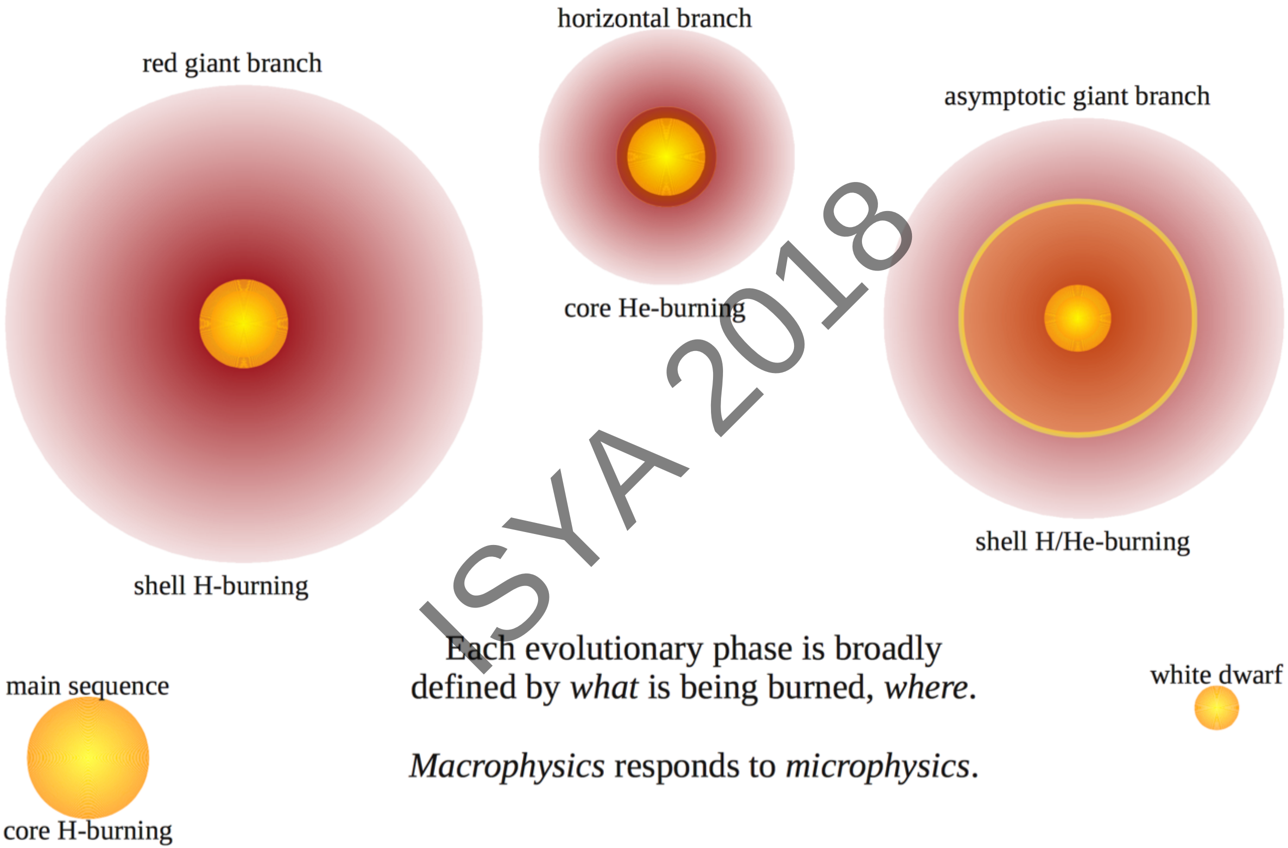
Class 4 Review:

Hayashi lines → Pre-Main Sequence → Zero-age Main Sequence ZAMS → Main Sequence MS Hydrogen burning

→ Post Main Sequence → Red Giant Branch RGB → Horizontal Branch He burning → Asymptotic Giant Branch AGB

- Changes in the fraction of elements
- Changes in the "boundaries" and nature of burning shells
- Use of the turn-off point to date clusters.

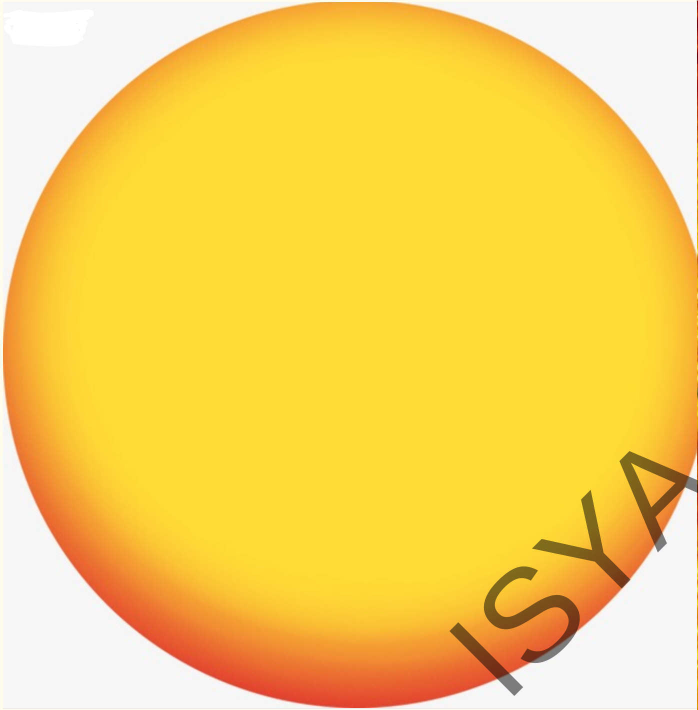
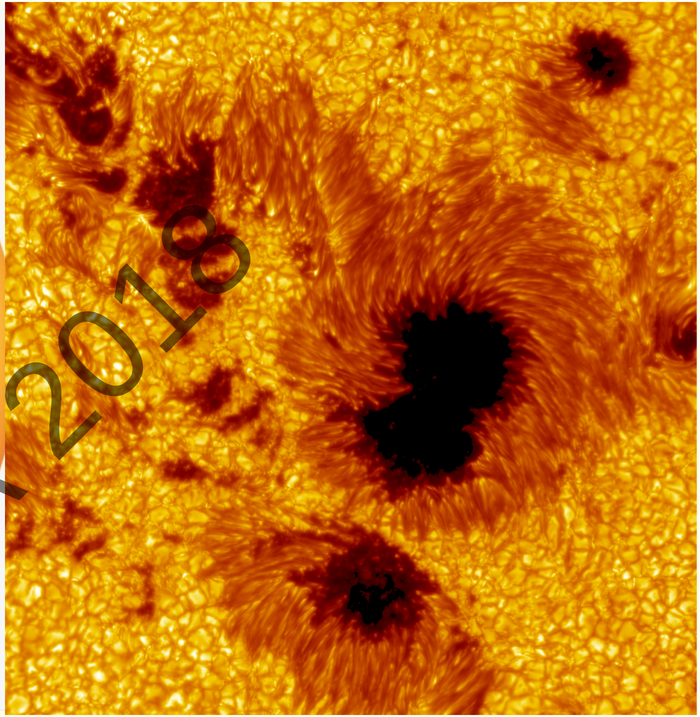
Class 4 Review:



Each evolutionary phase is broadly defined by *what* is being burned, *where*.

Macrophysics responds to *microphysics*.

Warwick Ball

Stellar Atmospheres → Surface Layers → Observable Spectrum
(Photosphere)
(Chromosphere)

Thickness:

Sun $\Delta h \simeq 1000\,\text{km} \ll R_\odot$

White Dwarf $\Delta h \leq 100\,\text{m}$

Plane-parallel approximation

Red super giant $\Delta h/R \simeq 1$ → spherical models

But... Spots
Rotation
Binarity
$\underset{\smile}{B}$
Winds

Inhomogeneous surface!

Neglected or modelled locally

Modelling atmospheres:

Solve a set
2D integro-differential equations of I_ν → $f(\text{depth}, \text{angle})$

Photosphere and Chromosphere: Continuum radiation magneto-hydrodynamics

Corona and winds: Specific conditions

ϵ transport: radiation (entire atmosphere)
convection (cool atmosphere)
conduction (e^-, corona)

Forces involved: $\underset{\to}{F}_{grav}$, $\underset{\to}{P}_{rad}$, $\underset{\to}{P}_{gas}$, $\underset{\to}{F}_{Lorentz}$

Matter mixing: gas motion
dust clouds ← brown dwarf "hot topic"

$$\frac{\partial \rho}{\partial t} = \underline{v}\nabla\rho - \rho\nabla\cdot\underline{v}$$

Mass conservation

viscous stress tensor

$$\rho\frac{\partial \underline{v}}{\partial t} = -\rho\underline{v}\cdot\nabla\underline{v} + \rho\underline{g} - \nabla P + \nabla\cdot\tau + \underline{F}_{Lorentz} + \underline{P}_{rad}$$

Momentum conservation

$$\rho\frac{\partial \epsilon}{\partial t} = -\rho\underline{v}\nabla\epsilon - P\nabla\cdot\underline{v} + \rho\left(Q_{visc} + Q_{Joule} + Q_{rad}\right)$$

Energy conservation

electric resistivity

$$\frac{\partial \underline{B}}{\partial t} = \nabla\times(\underline{v}\times\underline{B}) - \nabla\times(\eta\nabla\times\underline{B})$$

Induction equation

$$P = P(\rho, \epsilon)$$

Equation of state

Radiative MHD equations

shorcut:

Basic Assumption: A. Isolation

Limited spatial resolution
Limited time resolution
(finite t_{exp})

1. § local thermal equilibrium: each absortion is linked with an emission at every radii.
   Unrealistic at the surface.

2. § local hydrostatic equilibrium: $\vec{v} = 0$
   $\tau_{dynamical} \ll \tau_{KH}$
   Possible stationary winds

3. Not angular dependence $\longrightarrow$ plane atmosphere $\theta \to \parallel$
   1D geometry

4. $\vec{B} = 0$, well mixed

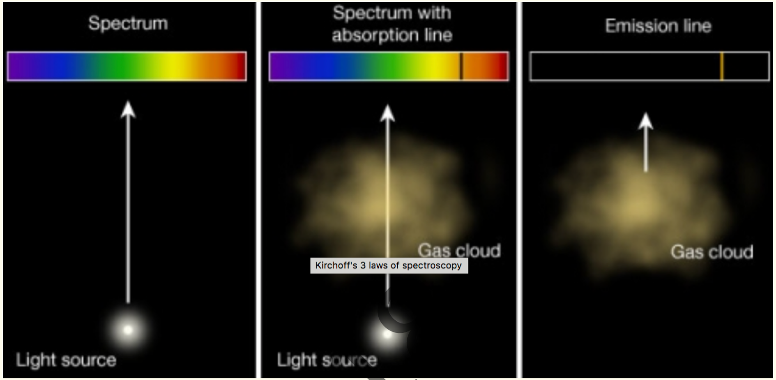
Iterative models: temperature correction

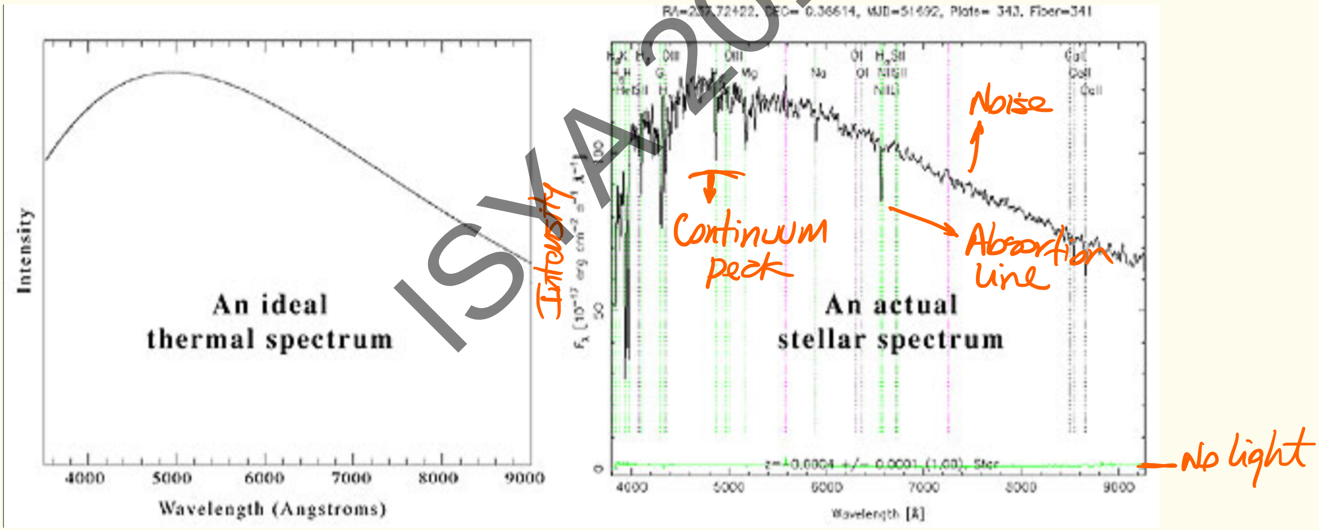
- Effective temperature $\longrightarrow$ $F_{tot} = \sigma T_{eff}^4$
- Surface gravity $\longrightarrow$ $\log g$ $\longleftarrow$ extremelly important for young sources
- Abundances $\longrightarrow$ Chemical Evolution
- Radial velocity $\longrightarrow$ Kinematics

L, R, M

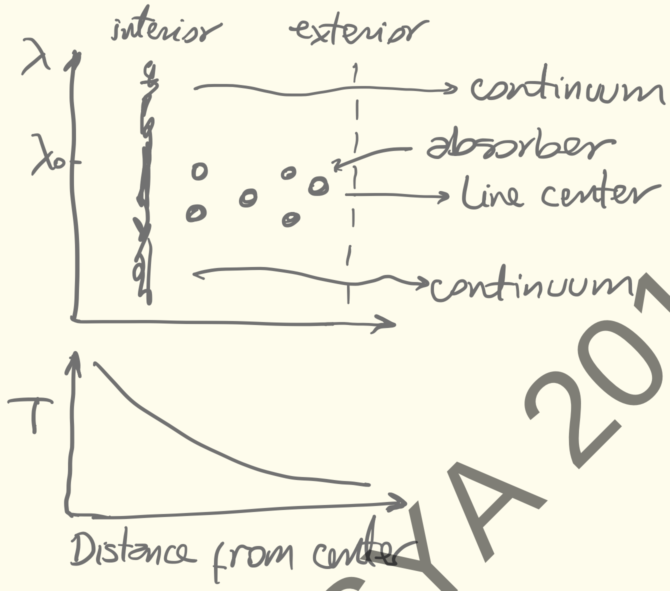
Star	Spec. Type	$T_{\rm eff}$ [K]	$\log g$ [cm/s^2]	Radius [10^3km]
Sun	G2V	5 780	4.44	696
Procyon	F5IV	6 500	4.0	1 500
White Dwarf	DA	12 000	8.0	0.0075
Red Giant	KII-III	3 700	1.0	66 000
Brown Dwarf	L	1 500	5.0	70

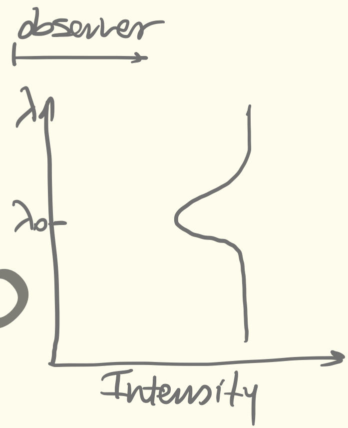
Line formation:





Line formation:





- Line continuum $\longrightarrow$ deepest layers of the photosphere.
- Part of emitted light is intercepted by absorbing atoms in the upper atmosphere.

Line strength:

Absorption = Pure (thermal) absorption κ_ν + Scattering σ_ν

Pure (thermal) absorption:
- Atom ionization
- Electronic transitions
- Molecule disociation
- Free-free absorption

Scattering:
- Thomson scattering (Free e^-)
- Rayleigh scattering (Molecules)
- Compton scattering (due to e^- rel)

- As T increases in the atmosphere it is possible to ionize more complex atoms by the energetic photons.

  $0.75\ eV \rightarrow H^-$ $\quad$ $7.90\ eV \rightarrow Fe$

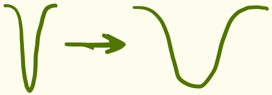
- Molecule disociation contributes to the continuum.
- Different absorption scenarios are related with gas density.
- Absorbed energy → retained energy → gas temperature increases → <u>Blanketing</u>

Energy transition Probabilities → line Absorption coefficient → Lorentz Profile → Natural line width

f_{lu}

K_ν^{line} , σ_ν^{line}

Line Widening:



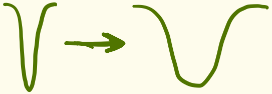
→ Natural: Uncertainty in the line generation. Not on unique ν. Uncertainty of level energy (Heisenberg's principle)

→ Pressure: if pressure increases the balance with gravity change in the atmosphere. Interaction with neighbouring particles (perturbers)

→ Collisional: Due to encounters of ions and atoms.

Line Widening:

→ Rotational: Disk moves differently in each point



→ Doppler: if T changes, E_{kin} changes and the are random movements → line ν → line $\Delta\nu$

$\phi(\Delta\nu)$

Real life Line → Vought Profile = Lorentz profile + Doppler Widening

Equivalent Width { Line Intensity $W_\lambda = \int \frac{I_{cont} - I_\lambda}{I_{cont}} \, d\lambda$

Atmosphere model → Comparison with observations → Observations → Parameter tunning (T_{eff}, g, X, Y, Z) → Atmosphere model

Spectral Classification:

Boltzmann equation: Number of atoms $N_{a,b}$ in a given excitation state with $E_{a,b}$ and degeneracy $g_{a,b}$

Relative population energy levels

$$\frac{N_b}{N_a} = \frac{g_b e^{-E_b/KT}}{g_a e^{-E_a/KT}} = \frac{g_b}{g_a} e^{-(E_b - E_a)/KT}$$

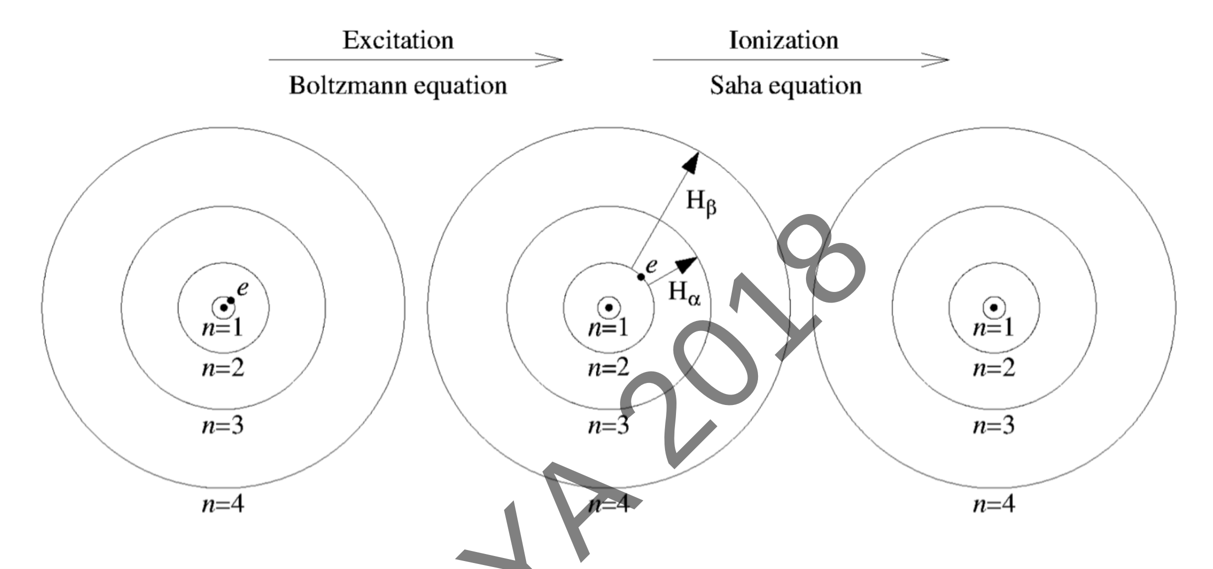
Saha equation: Number of atoms $N_{i,i+1}$ in a given ionization state with energy X_i

Relative population of ions

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i}\left(\frac{2\pi m_e k T}{h^2}\right)^{3/2} e^{-X_i/KT}$$

$$Z = \sum_{j=1}^{\infty} g_j e^{-(E_j - E_1)/KT}$$

Partition function: sum of the number of ways to arrange the atomic electrons.



Hydrogen T<9900K

Hydrogen T=9900K
e^- excited
Production of Balmer lines

Hydrogen T>9900K
unbound e^-
Ionized H

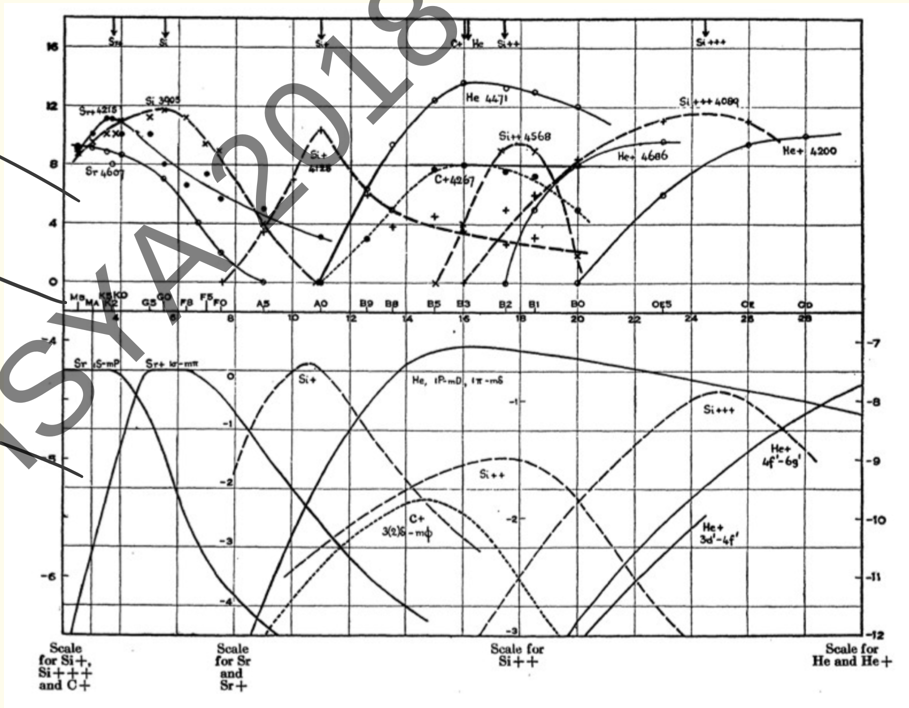
Saha Equation + Boltzmann Equation = Number densities of any atom in all excitation levels and ionization states in a gas at thermal equilibrium

Observed line strengths

Spectral Classification

Temperature Scale

Saha-Boltzmann Predictions



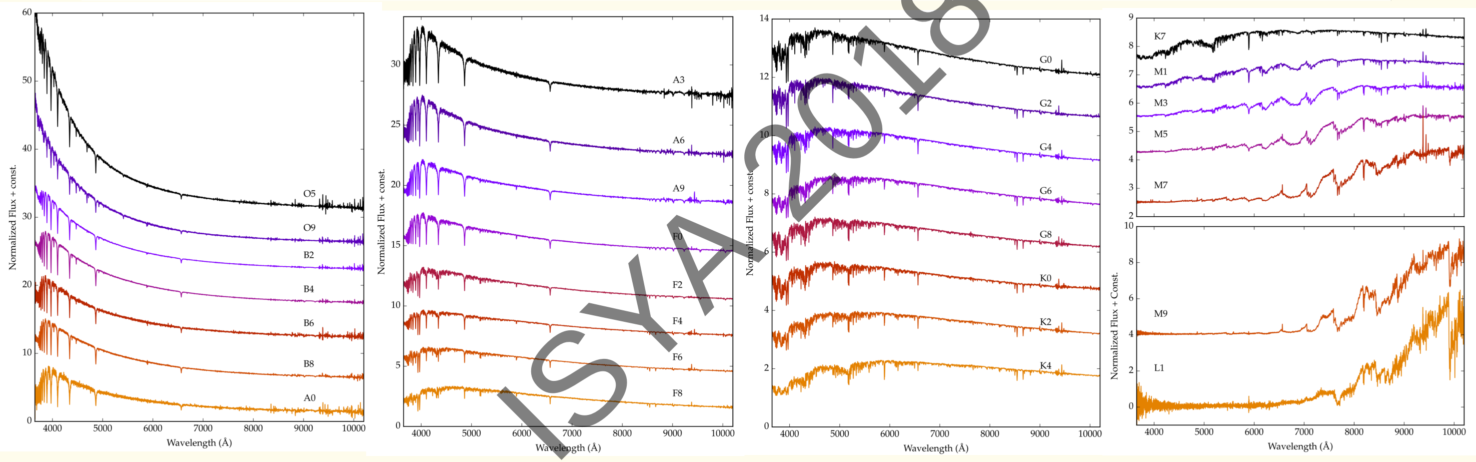
Payne 1924

The Astrophysical Journal Supplement Series, 230:16 (21pp), 2017 June
https://doi.org/10.3847/1538-4365/aa656d

CrossMark

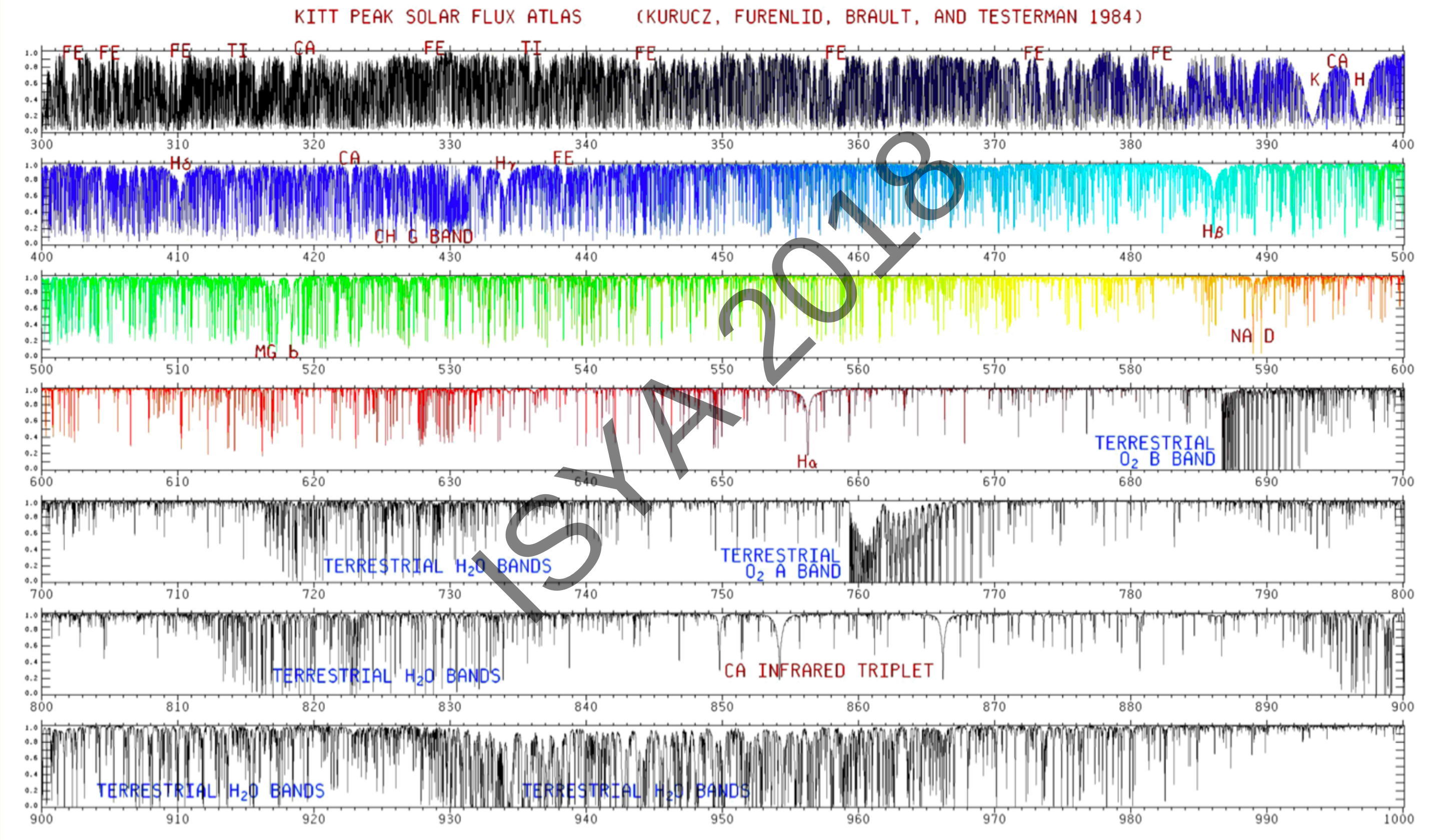
An Empirical Template Library of Stellar Spectra for a Wide Range of Spectral Classes, Luminosity Classes, and Metallicities Using SDSS BOSS Spectra

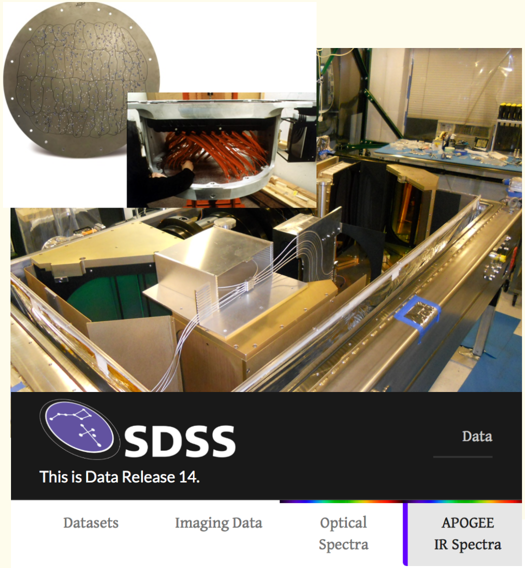
Aurora Y. Kesseli[1], Andrew A. West[1], Mark Veyette[1], Brandon Harrison[1], Dan Feldman[1], and John J. Bochanski[2]
[1] Boston University Astronomy Department, 725 Commonwealth Ave., Boston, MA 02215, USA; aurorak@bu.edu
[2] Rider University, 2083 Lawrenceville Rd., Lawrence Township, NJ 08648, USA
Received 2016 September 21; revised 2017 February 13; accepted 2017 February 13; published 2017 June 6

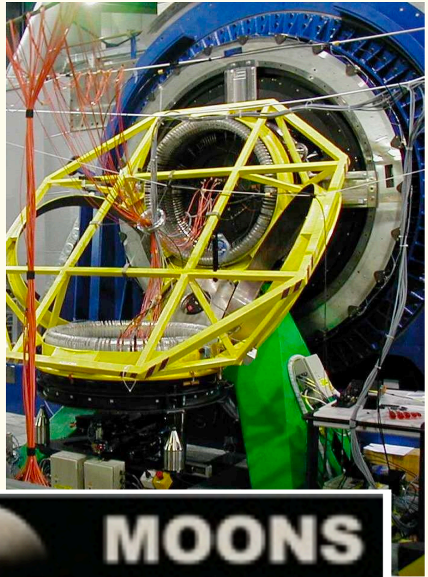


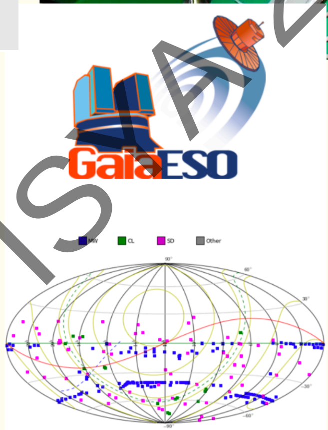
Dwarfs and Giants
Metallicity bins

Model Atmospheres




SDSS
Data
This is Data Release 14.
Datasets
Imaging Data
Optical Spectra
APOGEE IR Spectra




GaiaESO
CL
SD
Other


MOONS
Multi Object Optical and
Near-infrared Spectrograph

Sun (Fundamental Parameters) → Fundamental Stellar Structure Equations → • Equation of state (Non)classical (Non) Relativistic —

• Element conversion

→ Simplified stellar Interiors Models → Stellar core Evolution → Descriptive (Sub)stellar Evolution —

→ Stellar Atmospheres

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