The Suyama-Yamaguchi Consistency Relation and its Application to Cosmological Inflationary Models Involving Vector Fields

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Why studying models with vector fields?

- Vector field perturbations may cause part of the primordial curvature perturbation.
- Vector fields are natural sources for statistical anisotropies in the distribution of primordial fluctuations.
- The power spectrum and the bispectrum of ζ can become scale and shape dependent (configuration of wavevectors in momentum space).
- In presence of vector fields, non-Gaussianities and statistical anisotropies could be related.
- Anisotropy parameters can be very useful tools to discriminate among inflationary models.
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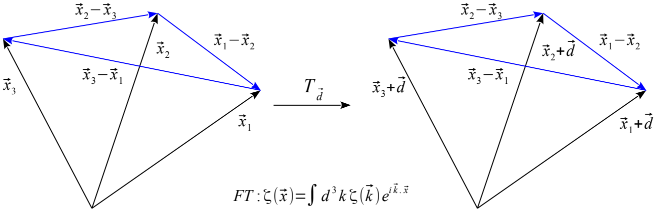
Statistical homogeneity

$$\langle \zeta(\vec{x}_1)\zeta(\vec{x}_2)\ldots\zeta(\vec{x}_n)\rangle \xrightarrow{T_{\vec{d}}} \langle \zeta(\vec{x}_1+\vec{d})\zeta(\vec{x}_2+\vec{d})\ldots\zeta(\vec{x}_n+\vec{d})\rangle$$

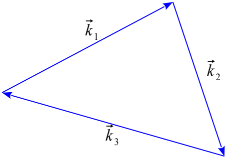


$$FT : \zeta(\vec{x}) = \int d^3k\,\zeta(\vec{k})\,e^{i\vec{k}.\vec{x}}$$

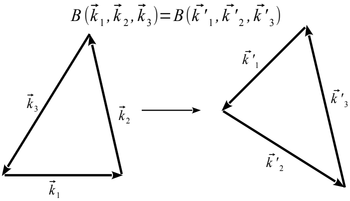
$$\downarrow$$

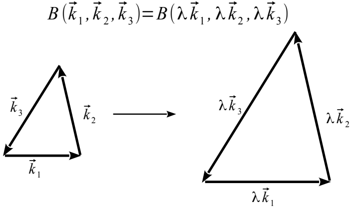
$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\ldots\zeta(\vec{k}_n)\rangle = \delta(\vec{k}_{12\ldots n})F_\zeta(\vec{k}_1,\vec{k}_2,\ldots,\vec{k}_n)$$

$$\vec{k}_1+\vec{k}_2+\vec{k}_3=0$$



Statistical isotropy and scale invariance



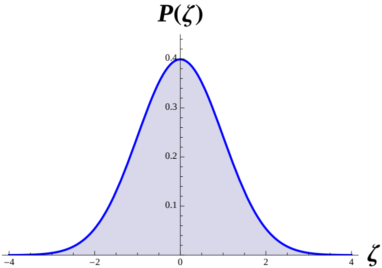


Gaussianity

$$\mathrm{P}[\zeta(\vec{x})] = \frac{1}{\sqrt{2\pi\langle\zeta^2(\vec{x})\rangle}} e^{-\zeta^2(\vec{x})/2\langle\zeta^2(\vec{x})\rangle}$$

The PDF is fully described with the two-point correlator, the *power spectrum* (PS):

$$\begin{aligned}\langle\zeta_{\vec{k}}\zeta_{\vec{k}'}\rangle &= (2\pi)^3\delta(\vec{k}+\vec{k}')P_\zeta(k) \\ &= \delta(\vec{k}+\vec{k}')\frac{2\pi^2}{k^3}A_\zeta\left(\frac{k}{k_0}\right)^{n_\zeta-1}\end{aligned}$$



Statistical Descriptors for ζ

If the PDF is anisotropic and Non-Gaussian, we

$$\text{SD:}\begin{cases} \text{Spectrum} & P_\zeta \begin{cases} \text{Amplitude } A_\zeta, \\ \text{Spectral Index } n_\zeta, \\ \text{The level of statistical anisotropy } g_\zeta. \end{cases} \\ \text{Bispectrum} & B_\zeta \begin{cases} \text{Products of the spectrum } P_\zeta, \\ \text{The level of non}-\text{gaussianity } f_{\rm NL}. \end{cases} \\ \text{Trispectrum} & T_\zeta \begin{cases} \text{Products of the spectrum } P_\zeta, \\ \text{The level of non}-\text{gaussianity } \tau_{\rm NL}, \\ \text{The level of non}-\text{gaussianity } g_{\rm NL}. \end{cases} \end{cases}$$

$$P_\zeta(\vec{k}) = P_\zeta^{\rm iso}\left[1 + g_\zeta(\hat{k}\cdot\hat{n})^2\right]$$

$$B_\zeta(\vec{k}_1, \vec{k}_2, \vec{k}_3) = B_\zeta^{\rm iso}\left[1 + g_\zeta b_1(\hat{k}_i, \hat{n}) + g_\zeta^2 b_2(\hat{k}_i, \hat{n})\right]$$

$$T_\zeta(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) = T_\zeta^{\rm iso}\left[1 + g_\zeta t_1(\hat{k}_i, \hat{k}_{jk}, \hat{n}) + g_\zeta^2 t_2(\hat{k}_i, \hat{k}_{jk}, \hat{n}) + g_\zeta^3 t_3(\hat{k}_i, \hat{k}_{jk}, \hat{n})\right]$$

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NG parameters in the BS and the TS

$$\frac{6}{5} f_{\rm NL} = \frac{\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle}{P_\zeta(k_1) P_\zeta(k_2) + 2\text{perm.}}$$

$$\tau_{\rm NL} = \frac{\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle}{P_\zeta(k_1) P_\zeta(k_2) P_\zeta(k_{12}) + 11\text{perm.}}$$

$$g_{\rm NL} = \frac{25}{54} \frac{\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle}{P_\zeta(k_1) P_\zeta(k_2) P_\zeta(k_3) + 3\text{perm.}}$$

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- Consistency relations offer a useful way to classify different inflationary models depending on their level of non-Gaussianity. They provide powerful criteria to rule out inflationary models.
- In vector field models, consistency relations between non-Gaussian parameters and the level of statistical anisotropy could appear (*"Anisotropic Non-Gaussianity"*) . NG measurements can be used to constrain statistical anisotropies measurements and vice-versa!
- Consistency relations could be used to constrain the parameters related to different effects with vector field models such as parity violations, anisotropic inflation, loop corrections, etc.
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The Suyama-Yamaguchi (SY) consistency relations.
The SY consistency relation, relates two of the statistical descriptors (SD) for the primordial curvature perturbation ζ, namely the levels of non-gaussianity in the bispectrum $f_{\rm NL}$ and in the trispectrum $\tau_{\rm NL}$.

First Variety

$$\tau_{\rm NL} \geqslant \left(\frac{6}{5} f_{\rm NL}\right)^2$$

Conditions

- *Condition 1*: The calculation of $f_{\rm NL}$ and $\tau_{\rm NL}$ is performed at tree level in the diagrammatic approach of the δN formalism.
- *Condition 2*: The inflationary dynamics is driven by any number of slowly-rolling scalar fields.
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- *Condition 4*: $f_{\rm NL}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ is evaluated in the squeezed limit ($\mathbf{k}_1 \to 0$) while $\tau_{\rm NL}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)$ is evaluated in the collapsed limit ($\mathbf{k}_1+\mathbf{k}_2 \to 0$).

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Third Variety

$$\int \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_3}{(2\pi)^6} \tau_{\rm NL}(\mathbf{k}_1,\mathbf{k}_3) P_\zeta(\mathbf{k}_1) P_\zeta(\mathbf{k}_3) \geqslant \left[\frac{6}{5}\int \frac{d^3\mathbf{k}_2}{(2\pi)^3} f_{\rm NL}(\mathbf{k}_2) P_\zeta(\mathbf{k}_2)\right]^2$$

The conditions to this variety are the same as before.

Fourth variety

$$\tau_{\rm NL}(\mathbf{k}_1,\mathbf{k}_3) \geqslant \left(\frac{6}{5}\right)^2 f_{\rm NL}(\mathbf{k}_1) f_{\rm NL}(\mathbf{k}_3)$$

This is a direct generalization of the first variety when there is no scale-invariance and whose form is easily inspired from the second and third varieties.

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1. It is valid even if there is statistical anisotropy and even if there is strong scale dependence.
2. The only required condition is statistical homogeneity.
3. This is the first time this variety is reported in the literature.
4. An observed violation of this consistency relation would imply statistical inhomogeneity which, in turn, would imply the impossibility of comparing theory and observation, affecting the foundations on which cosmology is constructed as a science.

Fifth Variety

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1. It is valid even if there is statistical anisotropy and even if there is strong scale dependence.
2. The only required condition is statistical homogeneity.
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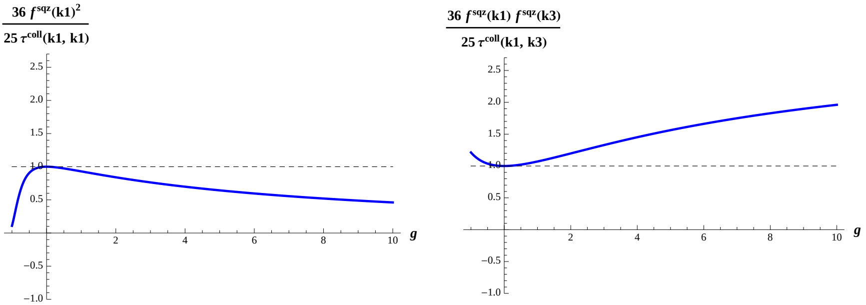
Results for the fourth and fifth variety



1. As expected, the fourth variety can be violated in several configurations of the momenta $\vec{k}_i$ and the vector $\hat{n}$.
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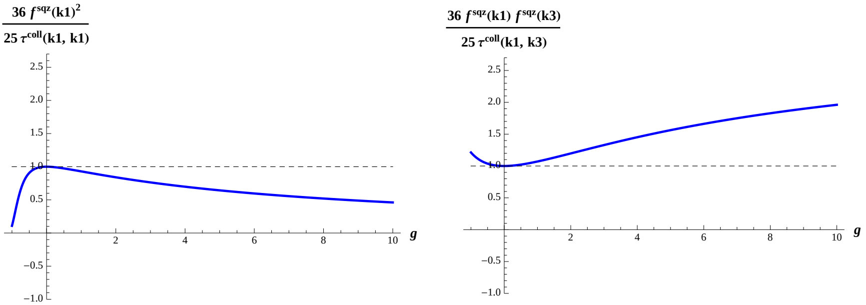

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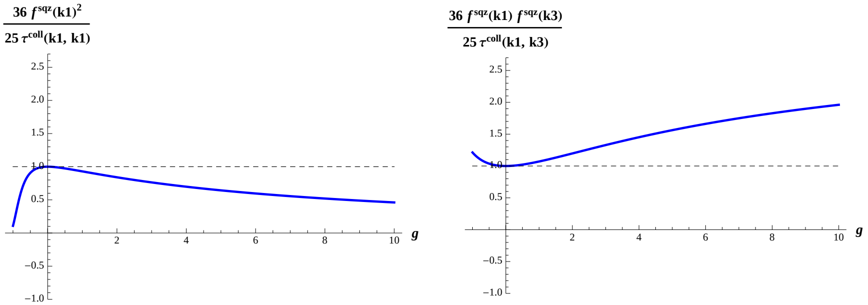

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Concluding remarks and possible improvements and interesting mechanisms to study

- The fifth variety of the SY consistency relation is a consequence of the fundamental assumption of statistical homogeneity.
- An observed violation of the fifth variety of the consistency relation would imply statistical inhomogeneity which, in turn, would imply the impossibility of comparing theory and observation,
  affecting the foundations on which cosmology is constructed as a science.
- One should be very careful about the definition of NG and anisotropy parameters used when compared with observations.
- Multi scalar and vector fields, even with correlations among them.
- Loop corrections seems to respect the same factorization of the scale dependent terms.
- Non Gaussianity of the fields at horizon crossing. (See e.g. Kehagias and Riotto hep-th 1205.1523v1 for the multi scalar case).
- Scale dependent PNG and statistical anisotropy.
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