ÍNDICE DE COMPRESIBILIDAD PARA ESFERAS ANISÓTROPAS EN RELATIVIDAD GENERAL

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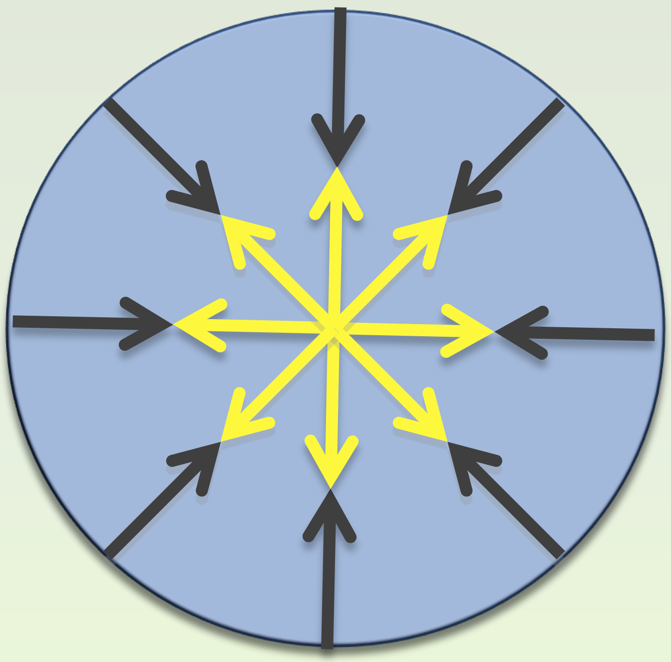
Problema de estudio

La resistencia de un material a ser comprimido (COMPRESIBILIDAD) es una variable fundamental en el estudio de Colapso Gravitacional. [1]

Índice de compresibilidad adiabático[1]:

$$\Gamma = \frac{\mathrm{d}\ln P}{\mathrm{d}\ln \rho}$$

Fuerzas Gravitacionales



Fuerzas Hidrodinámicas

(1) W. Barreto, L. Herrera and N. Santos. A generalization of the concept of adiabatic index for non-adiabatic systems. 1992Ap&SS.187..271B.

¿Cómo se está abordando?

Métrica de Schwarszchild:

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 d\Omega^2$$

$$T_r{}^{\mu}{}_{;\mu}$$

Tensor E-M adiabático:

$$T_{\mu}{}^{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -P_r & 0 & 0 \\ 0 & 0 & -P_{\perp} & 0 \\ 0 & 0 & 0 & -P_{\perp} \end{pmatrix}$$

Ecuación de equilibrio hidrostático:

$$\frac{\mathrm{d}P_r}{\mathrm{d}r} = -(\rho + P_r) \cdot \frac{m + 4\pi r^3 P_r}{r(r-2m)} + \frac{2}{r}(P_{\perp} - P_r)$$

Presión en función de la densidad de energía y la presión tangencial:

$$P = P(\rho, P_{\perp}, r)$$

Índice de compresibilidad

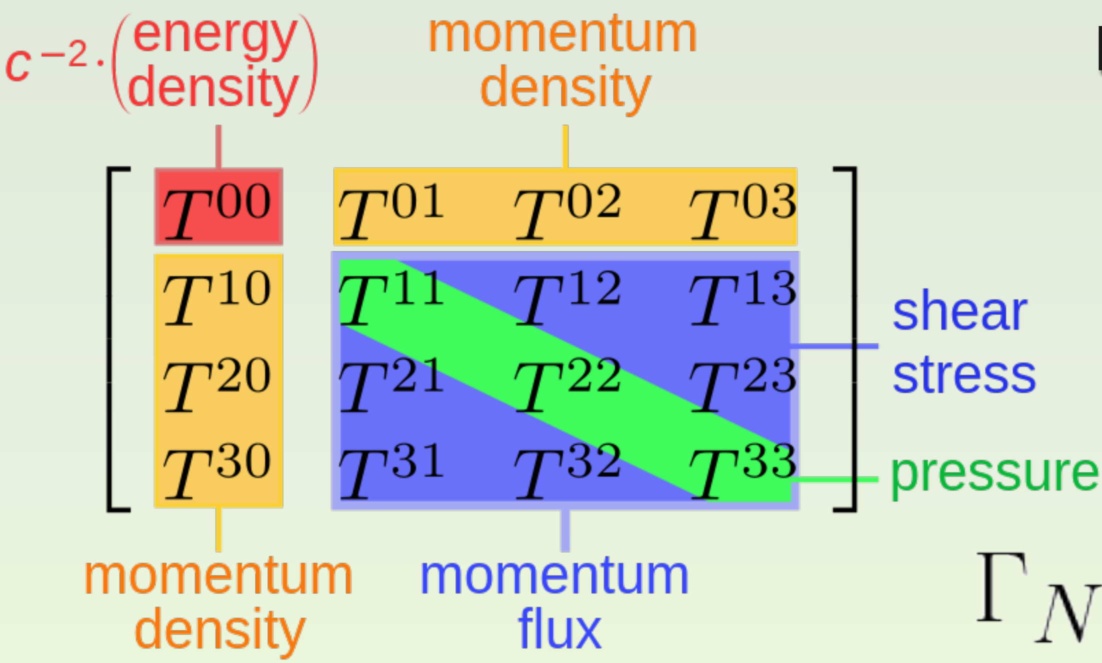
$$\Gamma = \frac{\rho}{P}\frac{\frac{\partial P}{\partial \rho}\frac{\mathrm{d}\rho}{\mathrm{d}r} + \frac{\partial P}{\partial P_\perp}\frac{\mathrm{d}P_\perp}{\mathrm{d}r} + \frac{\partial P}{\partial r}}{\frac{\mathrm{d}\rho}{\mathrm{d}r}}$$

$$\Gamma = \frac{\rho}{P}\left(\frac{\partial P}{\partial \rho} + \frac{\frac{\mathrm{d}\rho}{\mathrm{d}r} + \frac{\partial P}{\partial P_\perp}\frac{\mathrm{d}P_\perp}{\mathrm{d}r} + \frac{\partial P}{\partial r}}{\frac{\mathrm{d}\rho}{\mathrm{d}r}}\right)$$

ISÓTROPO ANISÓTROPO

A futuro…

Modelos no adiabáticos



Índice de compresibilidad no adiabático[1] y anisótropo

$$\Gamma_{N.A.} = \frac{\mathrm{d} \ln \Pi}{\mathrm{d} \ln E}$$

$$\Gamma_{N.A} = \frac{\mathrm{d} \ln \hat{T}^{\alpha 1} \hat{n}_{\alpha}}{\mathrm{d} \ln \hat{T}^{00}}$$

Imagen disponible en: http://commons.wikimedia.org/wiki/File:StressEnergyTensor_contravariant.svg

(1) W. Barreto, L. Herrera and N. Santos. A generalization of the concept of adiabatic index for non-adiabatic sistems. 1992Ap&SS.187..271B.

Estado actual del proyecto

Método de Cosenza, Esculpi y Witten(2)

$$P_{\perp}(r) - P_r(r) = C \cdot f\left(P_r(r), r\right) \cdot \left(\text{rho} + P_r(r)\right) \cdot r^n$$

$$f\left(P_r(r), r\right) \cdot r^{n-1} = \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}r} \mathrm{nu}(r);$$

$$P_{\perp}\left(P_r\right) = P_r + \frac{(1-h)}{2} \cdot \frac{\left(m(r) + 4 \cdot \mathrm{pi} \cdot r^3 \cdot P_r\right)}{(r - 2 \cdot m(r))} \cdot \left(\rho(r) + P_r\right)$$

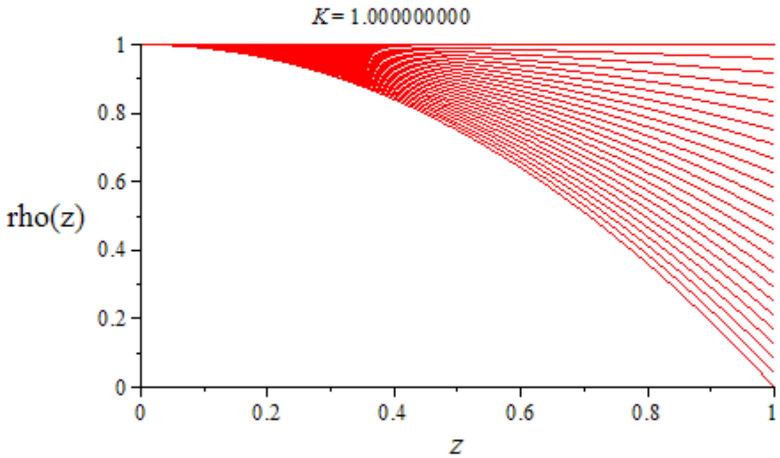
Tipo Tolman VI

$$\rho(r) = \frac{3}{56 \cdot \mathrm{pi} \cdot r^2}$$

$$P_r(r) = \frac{9 \cdot h}{56 \cdot \mathrm{pi} \cdot r^2} \cdot \left(\frac{1 - \left(\frac{r}{a}\right)^{\sqrt{4 - 3 \cdot h}}}{8 - 3 \cdot h + 4\sqrt{4 - 3 \cdot h} - \left(8 - 3 \cdot h - 4\sqrt{4 - 3 \cdot h}\right) \cdot \left(\frac{r}{a}\right)^{\sqrt{4 - 3 \cdot h}}} \right)$$

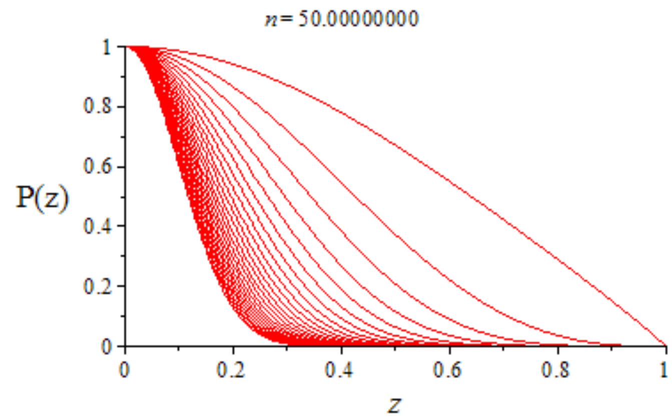
(2) M. Cosenza, L. Herrera, M. Esculpi, and L. Witten. Some models of anisotropic spheres in general relativity. Journal of Mathematical Physics, 22:118, 1981.

Método de Gokhroo y Mehra[3]



$$\rho(r) = \rho_0 \cdot \left(1 - \frac{K \cdot r^2}{a^2}\right)$$

$$P_r(r) = C \cdot \left(1 - \frac{2\left(-\frac{4}{5}\,\frac{\pi \rho_0 K r^5}{a^2} + \frac{4}{3}\,\pi \rho_0 r^3\right)}{r}\right) \cdot \left(1 - \frac{r^2}{a^2}\right)^n$$



$$P_{\perp}(r) = P_r(r) + \frac{4}{5}\,\frac{K C \pi \rho_0 r^4 \left(1 - \frac{r^2}{a^2}\right)^n}{a^2} + \frac{1}{2}\,\frac{r^2 \left(4 \pi C^2 \left(1 - \frac{8}{3}\pi \rho_0 r^2 + \frac{8}{5}\,\frac{K \pi \rho_0 r^4}{a^2}\right)^2 \left(1 - \frac{r^2}{a^2}\right)^{2n} - \frac{2 \pi C \left(1 - \frac{8}{3}\pi \rho_0 r^2 + \frac{8}{5}\,\frac{K \pi \rho_0 r^4}{a^2}\right)^2 \left(1 - \frac{r^2}{a^2}\right)^{n-1}}{a^2} + \frac{4}{3}\pi \rho_0^2 \left(1 - \frac{3}{5}\,\frac{K r^2}{a^2}\right)\left(1 - \frac{K r^2}{a^2}\right)\right)}{1 - \frac{8}{3}\pi \rho_0 r^2 + \frac{8}{5}\,\frac{K \pi \rho_0 r^4}{a^2}}$$

(3) M. K. Gokhroo and A. L. Mehra. Anisotropic spheres with variable energy density in general relativity. Gen. Rel. Grav., 26(1):75 - 84, 1994.

GRACIAS